Math 2001 - Assignment 12

Due November 21, 2025

(1) List the equivalence classes for these equivalence relations:
  (a) The relation $\sim$ on subsets A, B of $\{1,2,3\}$ where $A \sim B$ if $|A| = |B|$.
  (b) $R = \{(x,y) \in \mathbb{Z}\colon |x| = |y|\}$ on $\mathbb{Z}$

(2) Let $\sim$ be an equivalence relation on a set A, let $a, b \in A$. Let $[a]$ denote the equivalence class of a modulo $\sim$. Show that

$$a \not\sim b \text{ iff } [a] \cap [b] = \emptyset.$$

(3) Complete the proof of the following:

Theorem. Let $\{A_i \ : \ i \in I\}$ be a partition of a set A. Then

$$x \sim y \text{ if } \exists i \in I : x, y \in A_i$$

defines an equivalence relation on A with equivalence classes A_i for $i \in I$.

Proof: For reflexivity: Let $x \in A$. Since $A =$______________ by the definition of ______________, we have $i \in I$ such that $x \in$_____. Hence $x \sim$_____.

For ______________: Let $x, y \in A$. Assume $x \sim y$, that is, ____________ for some $i \in I$. Then $y, x \in A_i$ and __________.

For transitivity: Let __________. Assume $x \sim y$ and $y \sim z$. Then we have $i \in I$ such that ______________ and $j \in I$ such that ______________. Since _____$\in A_i \cap A_j$, we have __________ by the definition of a partition. Hence ____________ and $x \sim z$.

This completes the proof that $\sim$ is ______________.

Finally for every $x \in A$, the class $[x]_\sim =$ __________ for the unique $i \in I$ such that $x \in$ __________. ☐

(4) (a) Given finite sets A and B. How many different relations are there from A to B?
  (b) How many different equivalence relations are there on $A = \{1,2,3\}$? Describe them all by listing the partitions of A.

(5) (a) Give the tables for addition and multiplication for $\mathbb{Z}_6 = \mathbb{Z}/6\mathbb{Z}$.
  (b) Dividing by $[a]$ in $\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z}$ means solving an equation $[a] \cdot [x] = [1]$ for $[x]$.
  Solve $[8] \cdot [x] = [1]$ in $\mathbb{Z}_{37} = \mathbb{Z}/37\mathbb{Z}$.
  Hint: Use the Euclidean algorithm to solve $8x \equiv 1 \mod 37$.