Math 2001 - Assignment 11

Due November 14, 2025

(1) Let $p_1, p_2, \ldots$ denote the list of all primes. Show that for integers $a = \Pi_{i\in\mathbb{N}}\, p_i^{e_i}, b = \Pi_{i\in\mathbb{N}}\, p_i^{f_i}$ with $e_i, f_i \in \mathbb{N}_0$ for $i \in \mathbb{N}$,

$$\operatorname{lcm}(a,b) = \Pi_{i\in\mathbb{N}}\, p_i^{\max(e_i,f_i)}.$$

(2) Show for all $a, b \in \mathbb{N}$:

$$\gcd(a,b)\cdot \operatorname{lcm}(a,b) = ab$$

Hint: Use the formula for gcd and lcm from class and the previous problem.

Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive. Which are equivalences, which partial orders?

(3) $\neq$ on $\mathbb{Z}$
(4) $\subseteq$ on the power set $P(A)$ of a set A
(5) $|$ (divides) on $\mathbb{N}$
(6) $R = \{(x,y) \in \mathbb{R} \colon |x-y| \leq 1\}$