Math 2001 - Assignment 4

Due September 26, 2025

(1) (a) How many different truthtables (Boolean functions) are there for 2 statements x_1, x_2? How many for k statements $x_1, \ldots, x_k$?
(b) Let $f(x_1, x_2, x_3)$ be a Boolean function that is true for the following assignments and false otherwise.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	F	T
T	F	T	T
F	T	T	T

Write an expression for $f(x_1, x_2, x_3)$ using only $\wedge, \vee, \sim$.

(2) [1, Section 2.7]: Exercises 4,6,7,9,10. Also give the negation of the corresponding statements.

(3) Formulate the following sentences using quantifiers and logical operations. Are they true? Negate them.
(a) For all integers n we have that $n(n+1)$ is even.
(b) There exists a real number z such that $x + z = x$ for every real x.
(c) Every real number is smaller than some integer.

(4) Negate the following sentences. Are they true?
(a) If x^2 is rational, then so is x.
(b) $xy = 0$ iff $x = 0$ or $y = 0$
(c) The derivative of a polynomial function f is 0 iff f is constant.
(d) $\exists x \in \mathbb{R} : x^2 = -1$
(e) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer

(5) True or false? Give a proof or a counter-example:
(a) $\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}\ x + y = 1$
(b) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ x + y = 1$
(c) $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ x + y = 1$
(d) $\exists x \in \mathbb{R}\ \exists y \in \mathbb{R}\ x + y = 1$

(6) Write as complete English sentences. True or false? Negate.
(a) $\forall a \in \mathbb{R}\ \exists b \in \mathbb{R}\ \forall c \in \mathbb{R} :\ a < b \Rightarrow c < b$
(b) $\forall$ set A $\forall$ set B $\exists$ set C : $A \cup B = C$.
(c) $\forall x, y \in \mathbb{R}\ \forall \varepsilon > 0\ \exists \delta > 0 : |x - y| < \varepsilon \Rightarrow |2x - 2y| < \delta$

(7) Simplify:
(a) $\bigcup_{i=0}^{4} [i, 2i + 1]$
(b) $\bigcap_{n \in \mathbb{N}} \{x \in \mathbb{Z} : x \geq n\}$
(c) $\bigcup_{x \in [0,1]} \{x\} \times [1, 2]$
(d) $\bigcup_{x \in [0,1]} \{x\} \times [0, x]$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`