Math 2001 - Assignment 3

Due September 19, 2025

(1) Are the following statements? If so, determine whether they are true or false and write down their negation without the phrase "It is not true that ...".
   (a) Some swans are black.
   (b) Every real number is an even integer.
   (c) If x is an even integer, then $x + 1$ is odd.
   (d) $2x = 1$

(2) [1, Section 2.3]: Exercises 2,3,4,5,10

(3) Are the given statements true? Formulate their negations:
   (a) 2 is even, and 3 is even.
   (b) $2^n + 1$ is a prime number for every $n \in \mathbb{N}$.
   (c) There exists an even prime.
   (d) If the integer x is a multiple of 6, then x is even.

(4) Use truth tables to show that the following hold for all logical statements P, Q, R:
   (a) $P \vee (P \wedge Q) = P$
   (b) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

(5) Are the following equalities true for all statements P, Q?
   (a) $P \wedge \sim P =$ False
   (b) $\sim (P \Leftrightarrow Q) \;\; = \;\; \sim P \Leftrightarrow \sim Q$

(6) (a) How many different truthtables (Boolean functions) are there for 2 statements x_1, x_2? How many for k statements $x_1, \ldots, x_k$?
   (b) Let $f(x_1, x_2, x_3)$ be a Boolean function that is true for the following assignments and false otherwise.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	F	T
T	F	T	T
F	T	T	T

   Write an expression for $f(x_1, x_2, x_3)$ using only $\wedge, \vee, \sim$.

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`