Math 2001 - Assignment 2

Due September 12, 2025

(1) For $U := \{x \in \mathbb{Z} : 1 \leq x \leq 8\}$,
$A = \{1, 2, 3, 4, 5\}$,
$B = \{x \in U : x \text{ is even }\}$,
$C = \{x \in U : x \geq 4\}$ compute:
  (a) $A \cap \bar{B}$ (b) $A \cup (B \cap C)$ (c) $(A - B) \cup B$

(2) Are the following true for all sets A, B in a universe U?
  (a) $A - B = B - A$
  (b) $A \cup B \subseteq (A \cap \bar{B}) \cup (B \cap \bar{A})$

Consider Venn diagrams first and then either write a proof that the equations hold or give an example where they fail.

(3) Show that for all sets A, B, C

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

without Venn diagrams.

Recall that we already showed that the lefthand side is contained in the righthand side. So it only remains to write a proof for the converse,

$$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C).$$

(4) Show for all sets A, B in the universe U:

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \text{(de Morgan's law)}$$

First use Venn diagrams. Then write down a proof.

(5) Show that for all sets A, B in the universe U:

$$\overline{A} - \overline{B} = B - A$$

First consider Venn diagrams. Then write down the proof.

(6) Simplify the following sets and justify your answers:
  (a) $\bigcup_{n \in \mathbb{N}} (0, n]$ (b) $\bigcap_{n=1}^{3} \{nz : z \in \mathbb{Z}\}$ (c) $\bigcup_{A \in P(\mathbb{N})} A$

In (a) we have $(0, n] = \{x \in \mathbb{R} : 0 < x \leq n\}$, the real interval from 0 to n that does not contain 0 but contains n.

(7) Simplify the following sets and justify your answers:
  (a) $\bigcap_{n \in \mathbb{N}} \{nz : z \in \mathbb{Z}\}$ (b) $\bigcup_{x \in \mathbb{R}} [-x, x]$ (c) $\bigcap_{n \in \mathbb{N}} (-\frac{1}{n}, \frac{1}{n})$

In (c) we have $(-\frac{1}{n}, \frac{1}{n})$ the open interval not containing the end points.