Math 2001 - Assignment 1

Due September 5, 2025

(1) Element, subset or neither? Explain for each of the following whether $A \in B$ and $A \subseteq B$ are true or false:
(a) $A = 4, B = \mathbb{Z}$ (b) $A = \{1,2,3\}, B = \mathbb{Z}$
(c) $A = \{1,2\}, B = \{1,2,\{1,2\}\}$

Solution
(a) $4 \in \mathbb{Z}$ but $4 \not\subseteq \mathbb{Z}$
(b) $\{1,2,3\} \notin \mathbb{Z}$ but $\{1,2,3\} \subseteq \mathbb{Z}$
(c) $\{1,2\} \in \{1,2,\{1,2\}\}$ and $\{1,2\} \subseteq \{1,2,\{1,2\}\}$ since $1,2 \in \{1,2,\{1,2\}\}$

(2) Are the following true for $A = \{1,\{2,3\}\}$ or not?
(a) $\{2,3\} \in A$ (b) $\{2,3\} \subseteq A$ (c) $\emptyset \in A$ (d) $|A^2| = 9$.

Solution
(a) True (b) False (c) False (d) False, $|A^2| = 4$

(3) [1, Section 1.1]: Exercises 1,12,15

Solution
1. $\{5x - 1 : x \in \mathbb{Z}\} = \{\ldots, -6, -1, 4, 9, \ldots\}$
12. $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2,-1,0,1,2\}$
15. $\{5a + 2b : a, b \in \mathbb{Z}\} = \mathbb{Z}$

(4) Write each of the following sets using a defining property (Axiom of Specification) and using a function (Axiom of Replacement):
(a) $A = \{\ldots, -8, -4, 0, 4, 8, 12, \ldots\}$
(b) $B = \{0, 1, 2\}$
(c) $C =$ the set of even squares

Solution First specification, then replacement:
$A = \{x \in \mathbb{Z} \ : \ x \text{ is a multiple of } 4\} = \{4x \ : \ x \in \mathbb{Z}\}$
$B = \{x \in \mathbb{Z} \ : \ 0 \le x \le 2\} = \{x \ : \ x \in \{0,1,2\}\}$
$C = \{x \in \mathbb{Z} \ : \ x \text{ is an even square } \} = \{(2x)^2 \ : \ x \in \mathbb{Z}\}$

(5) [1, Section 1.1]: Exercises 29,38

Solution
29. $|\{\{1\}, \{2,\{3,4\}\}, \emptyset\}| = 3$
38. $|\{x \in \mathbb{N} : 5x \le 20\}| = |\{1,2,3,4\}| = 4$

(6) Let $A = \{0,1\}$ and $B = \{a,b,c\}$. Enumerate the elements of the following sets:
(a) $B \times A$ (b) $A \times \emptyset$ (c) A^3

Solution
(a) $B \times A = \{(a,0),(a,1),\dots,(c,1)\}$
(b) $B \times \emptyset = \emptyset$
(c) $A^3 = \{(0,0,0),\dots,(1,1,1)\}$ has 8 elements of triples with 0,1.

(7) Sketch the following Cartesian products in the plane $\mathbb{R}^2$. Be careful to denote whether the boundaries of your figures are contained in the sets or not (Use dashed lines for boundaries that are not included, solid lines for boundaries that are included).

(a) $\{1,2\} \times \{0,1,2\}$ (b) $[0,2] \times (1,2]$ (b) $\mathbb{R} \times \mathbb{Z}$

(8) Describe the following using either the Axiom of Specification or Replacement:
  (a) $A =$ the set of points in $\mathbb{R}^2$ on the line through $(2,3)$ that is parallel to the y-axis
  (b) $B =$ the set of points $(x,y) \in \mathbb{R}^2$ on the line through $(1,2)$ and $(3,4)$
  (c) $C =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0,0)$ and radius 2

Solution.
$A = \{(x,y) \in \mathbb{R}^2 \ : \ x = 2\} = \{(2,y) \ : \ y \in \mathbb{R}\}$
$B = \{(x,y) \in \mathbb{R}^2 \ : \ -x + y = 1\}$ by finding the equation of the line through $(1,2)$ and $(3,4)$
$C = \{(x,y) \in \mathbb{R}^2 \ : \ x^2 + y^2 = 4\}$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3nd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`