COMBINATORICS

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1. Combinatorial functions:

$n! := n \cdot (n-1) \cdots 2 \cdot 1$ is the number of permutations of $\{1, \ldots, n\}$.
$\binom{n}{k} := \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$

2. Counting the most common objects:

A k-tuple is a list of length k.
A k-subset is a set of size k.

Number of lists of length k with entries in $\{1, \ldots, n\}$

	with repetition	**without repetition**
order matters	n^k k-tuples	$\frac{n!}{(n-k)!}$
order does not matter	$\binom{k+n-1}{k}$ k-multisubsets	$\binom{n}{k}$ k-subsets

3. Inclusion-Exclusion:

For finite sets $A_1, \ldots, A_n$,

$$\begin{aligned}|A_1 \cup \ldots \cup A_n| = &|A_1| + \cdots + |A_n| \\ &- |A_1 \cap A_2| - |A_1 \cap A_3| - \cdots - |A_{n-1} \cap A_n| \\ &+ |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \cdots + |A_{n-2} \cap A_{n-1} \cap A_n| \\ &\vdots \\ &+ (-1)^{n-1} |A_1 \cap A_2 \cap \ldots \cap A_n|\end{aligned}$$

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