PUBLICATION LIST WITH ABSTRACTS

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1. Published and accepted papers

[1] **Marcos Mazari-Armida and Sebastien Vasey, *Universal classes near* $\aleph_1$, The Journal of Symbolic Logic 83 (2018), no. 4, 1633-1643.**

Shelah has provided sufficient conditions for an $L_{\omega_1,\omega}$-sentence ψ to have arbitrarily large models and for a Morley-like theorem to hold of ψ. These conditions involve structural and set-theoretic assumptions on all the $\aleph_n$'s. Using tools of Boney, Shelah, and the second author, we give assumptions on $\aleph_0$ and $\aleph_1$ which suffice when ψ is restricted to be universal:

Theorem. *Assume* $2^{\aleph_0} < 2^{\aleph_1}$. *Let* ψ *be a universal* $L_{\omega_1,\omega}$*-sentence.*

(1) If ψ *is categorical in* $\aleph_0$ *and* $1 \leq \mathbb{I}(\psi, \aleph_1) < 2^{\aleph_1}$*, then* ψ *has arbitrarily large models and categoricity of* ψ *in some uncountable cardinal implies categoricity of* ψ *in all uncountable cardinals.*
(2) If ψ *is categorical in* $\aleph_1$*, then* ψ *is categorical in all uncountable cardinals.*

The theorem generalizes to the framework of $L_{\omega_1,\omega}$-definable tame abstract elementary classes with primes.

[2] **Marcos Mazari-Armida, *Non-forking w-good frames*, Archive for Mathematical Logic 59 (2020), no. 1-2, 31-56.**

We introduce the notion of a w-good λ-frame which is a weakening of Shelah's notion of a good λ-frame. Existence of a w-good λ-frame implies existence of a model of size λ^{++}. Tameness and amalgamation imply extension of a w-good λ-frame to larger models. As an application we show:

Theorem 1.1. *Suppose* $2^{\lambda} < 2^{\lambda^+} < 2^{\lambda^{++}}$ *and* $2^{\lambda^+} > \lambda^{++}$. *If* $\mathbb{I}(\mathbf{K}, \lambda) = \mathbb{I}(\mathbf{K}, \lambda^+) = 1 \leq \mathbb{I}(\mathbf{K}, \lambda^{++}) < 2^{\lambda^{++}}$ *and* $\mathbf{K}$ *is* (λ, λ^+)*-tame, then* $\mathbf{K}_{\lambda^{+++}} \neq \emptyset$.

The proof presented clarifies some of the details of the main theorem of [Sh576] and avoids using the heavy set-theoretic machinery of [Sh:h, §VII] by replacing it with tameness.

[3] **Marcos Mazari-Armida, *Algebraic description of limit models in classes of abelian groups*, Annals of Pure and Applied Logic 171 (2020), no. 1, 102723.**

We study limit models in the class of abelian groups with the subgroup relation and in the class of torsion-free abelian groups with the pure subgroup relation. We show:

Theorem 1.2.

(1) If G is a limit model of cardinality λ in the class of abelian groups with the subgroup relation, then $G \cong (\oplus_\lambda \mathbb{Q}) \oplus \oplus_{p\ prime}(\oplus_\lambda \mathbb{Z}(p^\infty))$.

(2) If G is a limit model of cardinality λ in the class of torsion-free abelian groups with the pure subgroup relation, then:

- *If the length of the chain has uncountable cofinality, then*

$$G \cong (\oplus_\lambda \mathbb{Q}) \oplus \Pi_{p\ prime}\overline{(\oplus_\lambda \mathbb{Z}_{(p)})}.$$

- *If the length of the chain has countable cofinality, then G is not algebraically compact.*

We also study the class of finitely Butler groups with the pure subgroup relation, we show that it is an AEC, Galois-stable and $(< \aleph_0)$-tame and short.

[4] **Thomas G. Kucera and Marcos Mazari-Armida, *On universal modules with pure embeddings*, Mathematical Logic Quarterly 66 (2020), 395–408.**

We show that certain classes of modules have universal models with respect to pure embeddings.

Theorem 1.3. *Let R be a ring, T a first-order theory with an infinite model extending the theory of R-modules and $\mathbf{K}^T = (Mod(T), \leq_{pp})$ (where $\leq_{pp}$ stands for pure submodule). Assume $\mathbf{K}^T$ has joint embedding and amalgamation.*

If $\lambda^{|T|} = \lambda$ or $\forall \mu < \lambda(\mu^{|T|} < \lambda)$, then $\mathbf{K}^T$ has a universal model of cardinality λ.

As a special case we get a recent result of Shelah [She17, 1.2] concerning the existence of universal reduced torsion-free abelian groups with respect to pure embeddings.

We begin the study of limit models for classes of R-modules with joint embedding and amalgamation. We show that limit models with chains of long cofinality are pure-injective and we characterize limit models with chains of countable cofinality. This can be used to answer Question 4.25 of [Maz20].

As this paper is aimed at model theorists and algebraists an effort was made to provide the background for both.

[5] **Marcos Mazari-Armida, *Superstability, noetherian rings and pure-semisimple rings*, Annals of Pure and Applied Logic 172 (2021), no. 3, 102917.**

We uncover a connection between the model-theoretic notion of superstability and that of noetherian rings and pure-semisimple rings.

We characterize noetherian rings via superstability of the class of left modules with embeddings.

Theorem 1.4. *For a ring R the following are equivalent.*

(1) R is left noetherian.
(2) The class of left R-modules with embeddings is superstable.
(3) For every $\lambda \geq |R| + \aleph_0$, there is $\chi \geq \lambda$ such that the class of left R-modules with embeddings has uniqueness of limit models of cardinality χ.

If R is left coherent, they are further equivalent to:

(4) The class of left absolutely pure modules with embeddings is superstable.

We characterize left pure-semisimple rings via superstability of the class of left modules with pure embeddings.

Theorem 1.5. *For a ring R the following are equivalent.*

(1) R is left pure-semisimple.
(2) The class of left R-modules with pure embeddings is superstable.
(3) There exists $\lambda \geq (|R| + \aleph_0)^+$ such that the class of left R-modules with pure embeddings has uniqueness of limit models of cardinality λ.

Both equivalences provide evidence that the notion of superstability could shed light in the understanding of algebraic concepts.

As this paper is aimed at model theorists and algebraists an effort was made to provide the background for both.

[6] **Marcos Mazari-Armida, *On superstability in the class of flat modules and perfect rings*, Proceedings of AMS, 149 (2021), 2639–2654.**

We obtain a characterization of left perfect rings via superstability of the class of flat left modules with pure embeddings.

Theorem 1.6. *For a ring R the following are equivalent.*

(1) R is left perfect.
(2) The class of flat left R-modules with pure embeddings is superstable.
(3) There exists a $\lambda \geq (|R| + \aleph_0)^+$ such that the class of flat left R-modules with pure embeddings has uniqueness of limit models of cardinality λ.
(4) Every limit model in the class of flat left R-modules with pure embeddings is Σ-cotorsion.

A key step in our argument is the study of limit models in the class of flat modules. We show that limit models with chains of long cofinality are cotorsion and that limit models are elementarily equivalent.

We obtain a new characterization via limit models of the rings characterized in [Rot02]. We show that in these rings the equivalence between left perfect rings and superstability can be refined. We show that the results for these rings can be applied to extend [She17, 1.2] to classes of flat modules not axiomatizable in first-order logic.

[7] **Rami Grossberg and Marcos Mazari-Armida, *Simple-like independence relations in abstract elementary classes*, Annals of Pure and Applied Logic, 172 (2021), no. 7, 102971.**

We introduce and study simple and supersimple independence relations in the context of AECs with a monster model.

Theorem 1.7. *Let* $\mathbf{K}$ *be an AEC with a monster model.*

- *If* $\mathbf{K}$ *has a simple independence relation, then* $\mathbf{K}$ *does not have the* 2*-tree property.*
- *If* $\mathbf{K}$ *has a simple independence relation with the* $(< \aleph_0)$*-witness property for singletons, then* $\mathbf{K}$ *does not have the tree property.*

The proof of both facts is done by finding cardinal bounds to classes of small Galois-types over a fixed model that are inconsistent for large subsets. We think that this finer way of counting types is an interesting notion in itself.

We characterize supersimple independence relations by finiteness of the Lascar rank under locality assumptions on the independence relation.

[8] **Marcos Mazari-Armida, *A model theoretic solution to a problem of László Fuchs*, Journal of Algebra 567 (2021), 196–209.**

Problem 5.1 in page 181 of [Fuc15] asks to find the cardinals λ such that there is a universal abelian p-group for purity of cardinality λ, i.e., an abelian p-group U_λ of cardinality λ such that every abelian p-group of cardinality $\leq \lambda$ purely embeds in U_λ. In this paper we use ideas from the theory of abstract elementary classes to show:

Theorem 1.8. *Let* p *be a prime number. If* $\lambda^{\aleph_0} = \lambda$ *or* $\forall \mu < \lambda(\mu^{\aleph_0} < \lambda)$*, then there is a universal abelian* p*-group for purity of cardinality* λ*. Moreover for* $n \geq 2$*, there is a universal abelian* p*-group for purity of cardinality* $\aleph_n$ *if and only if* $2^{\aleph_0} \leq \aleph_n$*.*

As the theory of abstract elementary classes has barely been used to tackle algebraic questions, an effort was made to introduce this theory from an algebraic perspective.

[9] **Marcos Mazari-Armida, *Some stable non-elementary classes of modules*, Journal of Symbolic Logic, To appear, 25 pages.**
https://doi.org/10.1017/jsl.2021.68

Fisher and Baur showed independently in the seventies that for any T a complete first-order theory extending the theory of modules, $(Mod(T), \leq_p)$ is stable. In [Maz21, 2.12], it is asked if the same is true for any abstract elementary class $(K, \leq_p)$ such that K is a class of modules. In this paper we give some instances where this is true:

Theorem 1.9. *Let $(K, \leq_p)$ be an AEC of R-modules closed under finite direct sums, then:*

- *If K is closed under direct summands and pure-injective envelopes, then $\mathbf{K}$ is λ-stable for every λ such that $\lambda^{|R|+\aleph_0} = \lambda$.*
- *If K is closed under pure submodules and pushouts of pure embeddings, then $\mathbf{K}$ is λ-stable for every λ such that $\lambda^{|R|+\aleph_0} = \lambda$.*
- *Assume R is left Von Neumann regular. If $\mathbf{K}$ is closed under pure submodules, then $\mathbf{K}$ is λ-stable for every λ such that $\lambda^{|R|+\aleph_0} = \lambda$.*

As an application of these results we give new characterizations of noetherian rings, pure-semisimple rings, Dedekind domains and fields via superstability. Moreover, we show how these results can be used to show a link between being good in the stability hierarchy and being good in the axiomatizability hierarchy.

Another application is the existence of universal models with respect to pure embeddings in several classes of modules. Among them, the class of flat modules and the class of injective torsion modules.

2. Papers submitted for publication

[10] **Marcos Mazari-Armida, *A note on torsion modules with pure embeddings* , Submitted, 15 pages.**
https://arxiv.org/abs/2104.10160

We study Martsinkovsky-Russell torsion modules [MaRu20] with pure embeddings as an abstract elementary class. We give a model-theoretic characterization of the pure-injective and the Σ-pure-injective modules relative to the class of torsion modules assuming that the ring is right semihereditary. Our characterization of relative Σ-pure-injective modules extends the classical charactetization of [GrJe76] and [Zim77, 3.6].

We study the limit models of the class and determine when the class is superstable assuming that the ring is right semihereditary. As a corollary, we show that the class of torsion abelian groups with pure embeddings is strictly stable, i.e., stable not superstable.

[11] **Marcos Mazari-Armida, *Characterizing categoricity in several classes of modules* , Submitted, 14 pages.**
https://arxiv.org/abs/2202.07900

We show that the condition of being categorical in a tail of cardinals can be characterized algebraically for several classes of modules.

Theorem 2.1. *Assume R is an associative ring with unity.*

1. *The class of locally pure-injective R-modules is λ-categorical in* all $\lambda > |R| + \aleph_0$ *if and only if $R \cong M_n(D)$ for D a division ring and $n \geq 1$.*
2. *The class of flat R-modules is λ-categorical in* all $\lambda > |R|+\aleph_0$ *if and only if $R \cong M_n(k)$ for k a local ring such that its maximal ideal is left T-nilpotent and $n \geq 1$.*
3. *Assume R is a commutative ring. The class of absolutely pure R-modules is λ-categorical in* all $\lambda > |R| + \aleph_0$ *if and only if R is a local artinian ring.*

We show that in the above results it is enough to assume λ-categoricity in *some* big cardinal λ. This shows that Shelah's Categoricity Conjecture holds for the class of locally pure-injective modules, flat modules and absolutely pure modules. These classes are not first-order axiomatizable for arbitrary rings.

We provide rings such that the class of flat modules is categorical in a tail of cardinals but it is not first-order axiomatizable.

[12] **Ivo Herzog and Marcos Mazari-Armida, *A countable universal torsion abelian group for purity*, Submitted, 15 pages** https://arxiv.org/abs/2208.13913

We show that there is a countable universal abelian p-group for purity, i.e., a countable abelian p-group U such that every countable abelian p-group purely embeds in U. This is the last result needed to provide a complete solution to Problem 5.1 of [Fuc15] below $\aleph_\omega$. We introduce $\aleph_0$-strongly homogeneous p-groups, show that there is a universal abelian p-group for purity which is $\aleph_0$-strongly homogeneous, and completely characterize the countable $\aleph_0$-strongly homogeneous p-groups.

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