Math 2001 - Your Best Work Homework 4

On a previous assignment, you showed that $\sqrt{5}$ is irrational. The way most of you solved this was to prove a lemma that if $5|a^2$, then $5|a$. Most of you proved this lemma by contrapositive, treating 4 separate cases for the remainder when a is divided by 5. You can imagine using this same technique to prove $\sqrt{7}$ is irrational. That proof would require 6 cases, which is awkward but still manageable. And what if I now asked you to prove $\sqrt{97}$ is irrational? There would be 96 cases, which is disgusting, although still doable. If I next asked you to prove the square root of the prime number 27644437 is irrational, you be compelled to search for a better method that proves a more general result, rather than continuing to address individual numbers one at a time.

On this "best work" homework, you will prove that if p is prime, then $\sqrt{p}$ is irrational. This proof is significantly more involved than any other proof you have done, so you will receive many hints. On the course website, you can find a .tex file containing both an outline for the proof, and some steps done for you. Your job is to complete the proof.