Math 2001
Truth tables and logical equivalence

1. Write a truth table for the logical statement $\sim (P \vee Q) \implies (\sim P \wedge Q)$. Do it step by step (i.e. include columns for $\sim P$ and $P \vee Q$ etc., building up to the full expression).

2. Is the example in the previous problem true? (Hint: Is $P \wedge Q$ true? Is $P \vee Q$ true?)

3. Two statments are *logically equivalent* if they have the same truth values regardless of the input values of the variables. Name a much simpler expression that is logically equivalent to the expression in the first example.

4. Prove DeMorgan's Laws:
   - $\sim (P \wedge Q) = (\sim P) \vee (\sim Q)$,
   - $\sim (P \vee Q) = (\sim P) \wedge (\sim Q)$.

5. In Section 2.4 you saw that $P \Leftrightarrow Q$ is logically equivalent to $(P \implies Q) \wedge (Q \implies P)$. Thus, in some sense $\Leftrightarrow$ isn't needed – it can be 'generated' by $\implies$ and $\wedge$. Now find a way to generate $P \implies Q$ using only $\vee$, $\wedge$ and $\sim$. (This shows that in some sense $\Leftrightarrow$ isn't needed either!).

6. A *tautology* is a Boolean expression that evaluates to TRUE for all possible values of its variables. Work together (as always) to come up with an example of a tautology in two variables (you might try one variable first if you are stuck). Provide a proof (that is, a truth table) that it is a tautology.

7. A *contradiction* is a Boolean expression that evaluates to FALSE for all possible values of its variables. Come up with an example of a contradiction in two variables and prove that it is one.

8. How many lines (besides the header) does the truth table for a Boolean expression in 8 variables have?

9. How many logically distinct Boolean expression could you define on two variables? Writing out all possibilities is possible but a hassle. Instead figure out how to count the possibilities.

10. Can all of the boolean expressions on two variables be constructed using only $\wedge$, $\vee$, and $\sim$? In other words, are all boolean expressions on two variables logically equivalent to one that combines only $\wedge$, $\vee$, and $\sim$? Why or why not?

11. How many logically distinct Boolean expressions could you define on n variables?