

1. Without looking in your textbook, write precise and complete definitions for subset, union, intersection, difference and complement.

Let A and B be sets.

- We say A is a subset of B , and write $A \subseteq B$, if and only if every element of A is also an element of B .
- The union of A and B , denoted $A \cup B$, is the set $\{x: x \in A \text{ or } x \in B\}$.
- The intersection of A and B , denoted $A \cap B$, is the set $\{x: x \in A \text{ and } x \in B\}$.
- The difference of A and B , denoted $A - B$ or $A \setminus B$, is $\{x: x \in A \text{ and } x \notin B\}$.
- The complement of a set A with respect to a set U , denoted $\bar{A}$, is $U - A$.

2. Comment on this question: Find $\bar{A}$ if $A = \{2, 4, 6, \dots\}$.

First, it is not clear exactly which set A is.

If it is the set of even natural numbers, then

$\{x \in \mathbb{N}: x \text{ is even}\}$ is less ambiguous. Also, it is not clear with which set the complement is taken with respect to.

3. Suppose that $A_0 = \emptyset$, $A_1 = \{\emptyset\}$, $A_2 = A_1 \cup \{A_1\}$, $A_3 = A_2 \cup \{A_2\}$, $\dots$, $A_n = A_{n-1} \cup \{A_{n-1}\}$.

- (a) Write the sets A_2 , A_3 , and A_4 by listing their elements.

$$A_2 = \{\emptyset, \{\emptyset\}\}$$

$$A_3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$$

$$A_4 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$$

- (b) $|A_5| = 5$

$$(c) |A_n| = n$$

$$(d) A_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 = A_4$$

$$(e) A_0 \cap A_1 \cap A_2 \cap A_3 \cap A_4 = \emptyset$$

$$(f) A_4 - A_2 = \{ \{ \emptyset, \{ \emptyset \} \}, \{ \emptyset, \{ \emptyset \}, \{ \emptyset, \{ \emptyset \} \} \} \}$$

$$(g) |A_4 - A_1| = 4 - 1 = 3$$

$$(h) |A_{20} - A_{14}| = 20 - 14 = 6$$

$$(i) |\mathcal{P}(A_{10})| = 2^{|A_{10}|} = 2^{10}$$

4. Write down the precise and complete definition of cartesian product:

Let A and B be sets. The cartesian product of A and B , denoted $A \times B$, is the set $\{(a, b) : a \in A \text{ and } b \in B\}$.

5. Say $A = \{a, r, g\}$ and $B = \{x \in \mathbb{N} : x^2 < 5\} = \{1, 2\}$.

$$(a) |A \times B| = |\{(a, 1), (a, 2), (r, 1), (r, 2), (g, 1), (g, 2)\}| = 6$$

(b) Name an element in $A \times \mathcal{P}(A)$. $(a, \{a, r\})$

$$(c) |A \times \mathcal{P}(A)| = |A| \cdot |\mathcal{P}(A)| = |A| \cdot 2^{|A|} = 3 \cdot 2^3 = 24.$$

(d) Give an example of an element in the set $(A \times B) \times A$. $((a, 1), r)$

(e) Give an example of an element in the set $A \times (B \times A)$. $(a, (2, g))$

$$(f) A \times \emptyset = \emptyset$$

6. Write the following set by listing all of its elements:

$$\{\{0\}, 1\} \times \{\emptyset\} = \{(\{0\}, \emptyset), (1, \emptyset)\}.$$