RECURSIVE SEQUENCES.

- A SEQUENCE IS A (POSSIBLY INFINITE) COMMA-SEPARATED LIST OF NUMBERS.
- EX $2, 5, 10, 17, \underline{26}, \underline{37}, \underline{50}, \ldots$

  THIS SEQUENCE CAN BE DEFINED BY AN EXPLICIT FORMULA

  $a_n = n^2+1$
- A SEQUENCE IS A FUNCTION WHOSE DOMAIN IS $\mathbb{N}$
- A SEQUENCE CAN BE DEFINED RECURSIVELY. THIS MEANS ITS TERMS ARE DETERMINED FROM THE PREVIOUS ONES
- EX: $\begin{cases} b_1 = 4 \\ b_n = 5 + a_{n-1} \end{cases}$

  LIST OF THE FIRST FEW TERMS:

  $4, 9, 14, 19, \ldots$ This is an arithmetic sequence

  EXPLICIT FORMULA: $b_n = 4+5(n-1) = -1+5n$
- EX: $\begin{cases} c_1 = 2 \\ c_n = 3 \cdot c_{n-1} \end{cases}$

  LIST OF THE FIRST FEW TERMS:

  $2, 6, 18, 54, \ldots$ This is a geometric sequence

  EXPLICIT FORMULA: $c_n = 2 \cdot 3^{n-1}$
- HOW TO PROVE THESE FORMULAS ARE CORRECT:

  use mathematical induction
- EX: $\begin{cases} F_1 = F_2 = 1 \\ F_n = F_{n-1} + F_{n-2} \end{cases}$

  LIST OF THE FIRST FEW TERMS:

  $1, 1, 2, 3, 5, 8, \ldots$

  EXPLICIT FORMULA: $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$
- TO PROVE THE EXPLICIT FORMULA FOR THE FIBONACCI NUMBERS IS CORRECT, ORDINARY INDUCTION WON'T WORK. WE NEED STRONG INDUCTION.