Practice on Recursion, Induction and Strong Induction

Some of these problems can be done with Mathematical Induction, and some require Strong Induction.

1. Consider the recursive sequence defined by $g_1 = 5$, $g_n = 3 \cdot g_{n-1}$.
   (a) Write down the first 5 terms of the sequence.
   (b) Guess an explicit formula for the sequence.
   (c) Use induction to prove that your formula is correct. Note: your work will be clearer if part (b) you name your guess c_n. The proof is then to show that $g_n = c_n$.

2. Choose **one** of the following to write up. (You should be able to prove any of them):
   (a) Let a_n be the sequence defined by $a_1 = 1, a_2 = 8$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$.
      i. Write down the first 5 terms of the sequence
      ii. Prove that for all $n \in \mathbb{N}$, $a_n = 3 \cdot 2^{n-1} + 2(-1)^n$. (Again, most students have an easier time with this if they define $d_n = 3 \cdot 2^{n-1} + 2(-1)^n$, and show that $a_n = d_n$.)
   (b) Let b_n be the sequence defined by $b_1 = 1$, $b_2 = 2$, $b_3 = 3$, and $b_n = b_{n-1} + b_{n-2} + b_{n-3}$ for $n \geq 4$. This sequence is sometimes referred to as a Tribonacci sequence, with signature $(1, 2, 3)$. Show that $b_n \leq 3^n$.
   (c) Concerning the Fibonacci sequence, prove that $F_2 + F_4 + F_6 + \ldots + F_{2n} = F_{2n+1} - 1$.