PROOF BY STRONG INDUCTION

- VERY SIMILAR TO PROOF BY INDUCTION
- INSTEAD OF ASSUMING P_K AND PROVING P_{K+1} $(\forall K \in \mathbb{N})$, WE ASSUME $P_1, P_2, \ldots, P_K$ AND PROVE P_{K+1} $(\forall K \in \mathbb{N})$
- WE MAY NEED MORE THAN ONE BASE CASE
- TEMPLATE FOR PROOF BY STRONG INDUCTION:

> TO PROVE THE OPEN STATEMENT $P(n)$ IS TRUE $\forall n \in \mathbb{N}$:
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> STEP 1: PROVE $P(1)$ IS TRUE
> (OR PERHAPS THE FIRST SEVERAL $P(n)$ ARE TRUE)
>
> STEP 2:
>
> STEP 3: CONCLUDE $P(n)$ IS TRUE $\forall n \in \mathbb{N}$

EXAMPLE: SUPPOSE THERE ARE n TOWNS CALLED $t_1, t_2, \ldots t_n$. SAY EVERY PAIR OF THESE TOWNS ARE CONNECTED BY A ONE-WAY ROAD. SHOW THERE EXISTS A ROUTE THAT PASSES THROUGH EVERY TOWN.

STEP 1: WHEN $n=1$ (ONE TOWN), THE "EMPTY ROUTE" WORKS. WHEN $n=2$ (TWO TOWNS), THERE IS EITHER A ROUTE FROM $t_1 \to t_2$ OR VICE VERSA.

STEP 2: SUPPOSE ANY SET OF TOWNS WITH K TOWNS OR FEWER HAS A ROUTE PASSING THROUGH EVERY TOWN. NOW ADD A TOWN t_{K+1}. SPLIT $\{t_1, \ldots t_K\}$ INTO TWO SETS F AND T, WHERE F IS THE SET OF TOWNS WITH ROADS <u>TO</u> t_{K+1} AND T IS THE SET OF TOWNS WITH ROADS <u>FROM</u> t_{K+1}. BY ASSUMPTION, THERE IS A ROUTE r_1 PASSING THROUGH ALL TOWNS IN F, AND LIKEWISE A ROUTE r_2 PASSING THROUGH ALL TOWNS IN T. CREATE THE NEW ROUTE BY TAKING r_1, THEN GOING TO t_{K+1}, THEN TRAVERSING r_2. WE HAVE CREATED A ROUTE THROUGH ALL TOWNS.

STEP 3: THUS, A ROUTE EXISTS FOR ANY SET OF n TOWNS, WHERE n IS A NATURAL NUMBER.