PROOF BY MATHEMATICAL INDUCTION.

PROOF BY INDUCTION CAN BE THOUGHT OF LIKE DOMINOES.

KNOCK OVER THE FIRST DOMINO...
IT KNOCKS OVER THE SECOND ONE...
EACH DOMINO KNOCKS OVER THE NEXT...
THEY ALL FALL DOWN.

LET $P(n)$ BE AN OPEN SENTENCE, $n \in \mathbb{N}$. THE FIRST DOMINO REPRESENTS $P(1)$. THE SECOND DOMINO REPRESENTS $P(2)$, AND SO ON. KNOCKING THE FIRST DOMINO OVER REPRESENTS PROVING $P(1)$. KNOCKING THE SECOND DOMINO OVER REPRESENTS PROVING $P(2)$, AND SO ON. TO PROVE $P(n)$ IS TRUE $\forall n \in \mathbb{N}$, WE MUST SHOW

- THE FIRST DOMINO IS KNOCKED OVER
  SHOW $P(1)$ IS TRUE CALLED "THE BASE CASE", OR "ANCHORING THE INDUCTION."
- THE DOMINOES ARE SET UP CORRECTLY, SO EACH ONE KNOCKS THE NEXT ONE OVER.
  ASSUME $P(k)$ IS TRUE, FOR $k \in \mathbb{N}$, (CALLED THE INDUCTIVE HYPOTHESIS). SHOW $P(k+1)$ IS TRUE.

TEMPLATE FOR PROOF BY INDUCTION

PROPOSITION: $P(n)$ IS TRUE FOR ALL $n \in \mathbb{N}$

PROOF: (BY INDUCTION)

STEP 1: PROVE THE STATEMENT $P(1)$ IS TRUE

STEP 2: GIVEN ANY INTEGER $k \geq 1$, PROVE $P(k) \Rightarrow P(k+1)$
IN OTHER WORDS, ASSUME $P(k)$ AND USE IT TO PROVE $P(k+1)$

STEP 3: CONCLUDE $P(n)$ HOLDS FOR ALL $n \in \mathbb{N}$.

EXAMPLE. SHOW $1+2+3+\cdots+n=\frac{n(n+1)}{2}$. $(\star)$

NOTE: THE ABOVE "STATEMENT" REALLY REPRESENTS INFINITELY MANY STATEMENTS. IT TELLS YOU A FORMULA FOR THE SUM OF THE FIRST n INTEGERS, NO MATTER WHAT n IS.

PROOF: (BY INDUCTION), LET $P(n)$ BE THE OPEN STATEMENT $1+2+\cdots+n=\frac{n(n+1)}{2}$.

STEP 1: (BASE CASE)
WHEN $n=1$, THE LEFT-HAND SIDE OF $(\star)$ IS JUST 1. THE RIGHT-HAND SIDE IS $\frac{1(1+1)}{2}=1$.

SO $P(1)$ IS TRUE.

STEP 2: ASSUME $P(k)$ IS TRUE. THAT IS, ASSUME
$1+2+3+\cdots+k=\frac{k(k+1)}{2}$ (THIS IS THE INDUCTIVE HYPOTHESIS)

WE MUST SHOW $P(k+1)$. THAT IS, show
$$1+2+\cdots+(k+1)=\frac{(k+1)(k+1+1)}{2}.$$

NOW
$$\begin{aligned}
&1+2+\cdots+(k+1)\\
&=1+2+\cdots+k+(k+1)\\
&=(1+2+\cdots+k)+(k+1)\\
&=\frac{k(k+1)}{2}+(k+1) \quad \text{(By the inductive hypothesis)}\\
&=(k+1)\cdot\frac{k}{2}+(k+1)\cdot\frac{2}{2}\\
&=(k+1)\left(\frac{k+2}{2}\right)=\frac{(k+1)(k+1+1)}{2}.
\end{aligned}$$

STEP 3: WE SHOWED $P(1)$ IS TRUE AND THAT IF $P(k)$ IS TRUE THE $P(k+1)$ IS TRUE.
SO $P(n)$ IS TRUE FOR ALL $n\in\mathbb{N}$.
SO $1+2+\cdots+n=\frac{n(n+1)}{2}$ for all $n\in\mathbb{N}$.