SOLUTIONS

CHAPTER 5 – PROOF BY CONTRAPOSITIVE

RECALL THAT

$$P \Rightarrow Q \quad \text{IS LOGICALLY EQUIVALENT TO} \quad \sim Q \Rightarrow \sim P$$

DIRECT PROOF:

TO PROVE $P \Rightarrow Q$:

> PROOF:
> SUPPOSE P.
> ⋮
> THEREFORE Q. ■

PROOF BY CONTRAPOSITIVE:

TO PROVE $P \Rightarrow Q$:

> PROOF:
> SUPPOSE $\sim Q$.
> ⋮
> THEREFORE $\sim P$. ■

EX: IF a IS A MULTIPLE OF 14, THEN a IS NOT ODD.

- TO PROVE THIS DIRECTLY, ASSUME a is a multiple of 14, AND SHOW a is not odd.
- WRITE DOWN THE CONTRAPOSITIVE OF THE STATEMENT:
  If a is odd, then a is not a multiple of 14
- TO PROVE THE STATEMENT BY CONTRAPOSITIVE, ASSUME a is odd, AND SHOW a is not a multiple of 14

EX: PROVE THAT IF $x^2+3x<0$, THEN $x<0$.

PROOF (by contrapositive):

SUPPOSE $x \geq 0$.

Then $x+3 \geq 3 > 0$

so $x+3>0$

we see that $x+3$ and x are both non-negative. So their product is non-negative.

This means $x(x+3) \geq 0$,

or $x^2+3x \geq 0$

So if $x^2+3x<0$, then $x<0$.

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