MATH 2001
Proof by Contradiction

Our next style of proof is proof by contradiction.

The main idea: To prove a statement P, we start by assuming $\sim P$. Using logical reasoning, we search for, and find, two statements that stand in contradiction to each other. In other words, we show $C \wedge \sim C$.

The principle on which proof by contradiction is based:

Proof of this principle:

Template for proof by contradiction:

Example 1: Prove that for all integers a and b, $a^2 - 4b \neq 2$.

Example 2: Prove that there are no integers a, b for which $18a + 6b = 1$.

Using contradiction to prove conditional statements:
If we want to prove $P \implies Q$ using proof by contradiction, we should assume _____.
This can be rewritten as:

_____.

Template for proof by contradiction for a conditional statement:

Example 3: Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.