

Proposition: any amount of postage 14¢ or greater can be formed using 3¢ stamps and 8¢ stamps.

Proof 1 (induction)

Step 1: $14¢ = 1 \times 8¢ + 2 \cdot 3¢$

$$15¢ = 5 \times 3¢$$

Induction is anchored

Step 2: Say $k¢$ can be formed using 8¢ and 3¢ stamps, where $k \geq 15$. If k is formed including at least one 8¢ stamp (i.e., $k = 8a + 3b$, where $a, b \in \mathbb{N}$), then take away an 8¢ stamp and replace it with three 3¢ stamps, forming $(k+1)¢$.

(i.e., $k+1 = 8a + 3b + 1 = 8(a-1) + 3(b+3)$).

If k is formed with only 3¢ stamps, then there are at least 5 of them ($k \geq 15$), so remove five 3¢ stamps and replace it with two 8¢ stamps forming $k+1$. (i.e., $k = 3c$, $c \in \mathbb{N}$, $c \geq 5$, so $k+1 = 3(c-5) + 2 \cdot 8$).

Step 3: By induction, since if $k \geq 15$ and k can be formed with 8¢ and 3¢ stamps, then $k+1$ can be as well, we see n can be formed with 3¢ and 8¢ stamps, for any $n \in \mathbb{N}$.

Proof 2 (strong induction)

Step 1: $14¢ = 1 \times 8¢ + 2 \cdot 3¢$; $15¢ = 5 \times 3¢$; $16¢ = 2 \times 8¢$. So the

Induction is anchored.

Step 2: Suppose any amount of postage between 14 and k ($k \geq 16$) can be formed with 8¢ and 3¢ stamps. In particular, $k-2$ can be. Add one 3¢ stamp to the way we formed $k-2$, & thus we have formed $k+1$. (i.e., $k-2 = 8a + 3b$, where a, b are non-negative integers. Thus $k+1 = k-2 + 3 = 8a + 3(b+1)$).

Step 3: By strong induction I can form any amount of postage 14¢ or greater from 3¢ and 8¢ stamps.