Section 1.1 Notes

- what is a set?

  A set is a collection of objects (called elements). There are no duplicates and order doesn't matter.

- notation
  - Elements go within braces, comma-separated: $\{\text{☺}, \text{☹}, \text{😵}\}$
  - Sets made of infinite lists: $\{\ldots, -2, -1, 0, 1, 2, 3 \ldots\}$
  - Set membership: $a \in A$ means a is an element of set A.

    $\text{☺} \in \{\text{☺}, \text{☹}, \text{😵}\}$ is True

    $\text{😐} \notin \{\text{☺}, \text{☹}, \text{😵}\}$ is True

    $\phi \in \{\text{☺}, \text{☹}, \text{😵}\}$ is False

    $\phi \subseteq \{\text{☺}, \text{☹}, \text{😵}\}$ is True
  - Special symbols: $\mathbb{R}, \mathbb{N}, \mathbb{Q}, \phi$ and interval notation, such as $[-1, 1]$ or $(0, 72]$
  - set-builder notation, used to avoid listing all elements.

    $\{$ expression : rule $\}$

    "the set" — "of elements" — "such that" — "these conditions are true"

    The expression gives the elements.
    The rule gives restrictions. The rule must be a statement.

- Cardinality, or size

  The size of a set A, written $|A|$, is the number of elements in A.

  Example: $|\{\text{☺}, \text{☹}, \text{😵}\}| = 3$

  Example: $|\{m \in \mathbb{Z} : |m| < 2\}| = |\{-1, 0, 1\}| = 3$

- Sets can have sets as elements.

  EX: $\{\mathbb{R}, \mathbb{Q}\}$. Note: $|\{\mathbb{R}, \mathbb{Q}\}| = 2$.

  EX: $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ has 3 elements

  EX: $\frac{5}{3} \notin \{\mathbb{Q}\}$.