midterm 2 practice questions
answers to selected questions.

1. "If a human being has an identical twin, then that person has a family member with the same birthday."
False - one twin could be born late in the evening & the other in the wee hours of the next day.

converse:
If a person has a family member with the same birthday, then that person has an identical twin.
False - there's about a 1/365 chance of a child sharing a birthday with the mother.

contrapositive:
If a person does not have a family member with the same b-day, then they do not have an identical twin. (also false)

converse of contrapositive:
If a person does not have an identical twin, then they do not have a family member with the same birthday (also false)

negation:
There exists a human being with an identical twin, but that person has no family member with the same b-day.
(surely this has happened, right? yes. I googled it, the negation is true!)

2. There exists a number divisible by 3 but not divisible by 6. That is, $\exists x \in \mathbb{Z}, 3 \mid x \wedge 6 \nmid x$
(True, by the way, try $x=3$)

4a $\sim[(R \vee Q) \Rightarrow (P \wedge S)] = (R \vee Q) \wedge \sim(P \wedge S)$

$= R \vee Q \wedge (\sim P \vee \sim S)$

4b. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy \neq 1$

4c. The function $f(x)$ is not a trig function, or $f(x)$ is continuous.

(note: "but" means "and", with some added implicit opinion)

4d. There are more than 10 stars in the milky way with at least 5 planets.

4e. Theresa is taller than 5'11", and she does not play basketball

4f. There exists a person who is taller than 5'11" and who does not play basketball.

4g. There exists a non-measurable subset of the real numbers, but every function is Lebesgue-integrable.

4h. $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, \frac{y}{x} \notin \mathbb{N}$

4i. There is a novel longer that 300 pages that everyone in class will read.

5. There exists a positive real number ε such that for all positive real numbers δ, there exist real numbers x and y such that $|x-y| < \delta$ and $|f(x)-f(y)| \geq \varepsilon$.

6a. True

6b. False

6c. True (Good practice proof)

6d. False $6 \mid (2\cdot 3)$, but $6 \nmid 2$ and $6 \nmid 3$

6e. False ($5 \in \mathbb{N}$, $6 \in \mathbb{N}$, but $5-6=-1 \notin \mathbb{N}$

6f. False ($\frac{1}{2} \in \mathbb{Q}$, $0 \in \mathbb{Q}$, but $\frac{\frac{1}{2}}{0} \notin \mathbb{Q}$.)

9. By the divisibility tricks we discussed,

$4+3+2+5 = 14 \equiv 5 \bmod 9$, so $4325 \equiv 5 \bmod 9$.

$5+4+6+2 = 17 \equiv 8 \bmod 9$, so $5462 \equiv 8 \bmod 9$.

So $4325 \times 5462 \equiv 5 \cdot 8 \pmod 9 \equiv 40 \pmod 9$

$\equiv 4 \pmod 9$

10. $(56325)^{13} \equiv 5^{13} \bmod (10)$

$5^2 \equiv 25 \pmod{10} \equiv 5 \pmod{10}$

$5^4 \equiv 5^2 \pmod{10} \equiv 5 \pmod{10}$

$5^8 \equiv 5^2 \pmod{10} \equiv 5 \pmod{10}$

$5^{13} = 5^8 \cdot 5^4 \cdot 5^1 = 5 \cdot 5 \cdot 5 \pmod{10} \equiv 125 \pmod{10} \equiv 5 \pmod{}$

11. Since $3 + 10 \equiv 0 \pmod{13}$, 3 & 10 are additive inverses (mod 10).

By trial-and-error, since $9 \cdot 3 = 27 \equiv 1 \pmod{13}$, 9 and 3 are multiplicative inverses (mod 13)

12. It's 5:00, so in 173402 it should be 173407 o'clock (Zorblogian time).

$173407 = (170000 + 3400 + 7) \pmod{17}$. $17 \mid 170000$ and $17 \mid 3400$. So $173407 \equiv 7 \pmod{17}$

It is 7 o'clock.