MATH 2001

Some Review Problems for Exam 2

1. Consider the statement “If a human being has an identical twin, then that person has a family member with the same birthday”. Is it true? Write the converse of the statement, the contrapositive of the statement, the converse of the contrapositive of the statement, and the negation of the statement.

2. Negate the statement “If a number is divisible by 3, then it is divisible by 6”.

3. Consider the statement “For every natural number, you can find a natural number so that the sum of the two numbers is 20”. Write the statement in symbolic logic, negate it, then write your negated statement in an English sentence. Is the statement true, or is its negation true?

4. Fully negate each of the following statements.

   (a) $(R \vee Q) \Rightarrow (P \wedge S)$

   (b) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \cdot y = 1$.

   (c) “The function $f(x)$ is a trig function, but it is not continuous.”

   (d) “There are at most 10 stars in the milky-way with at least 5 planets”.

   (e) If Theresa is taller than 5’11”, then she is a basketball player.

   (f) If a person is taller than 5’11”, then they are a basketball player.

   (g) If there exists a non-measureable subset of the real numbers, then there is a function that is not Lebesgue-integrable.

   (h) There exists a natural number x such that if y is a natural number, then $\frac{y}{x}$ is a natural number.

   (i) If a novel has more than 300 pages, then someone in the class will not read it.

5. Say that $f(x)$ is a function defined on $\mathbb{R}$. Negate this statement: For every positive real number ϵ, there is a positive real number δ with the property that for every real number x and y, if $|x-y| < \delta$ then $|f(x) - f(y)| < \epsilon$. (Note: youare beign asked to negate the definition of f being *uniformly continuous*).

6. True or false. Assume a, b and p are integers.

   (a) If $a|b$ then $a|2b$.

   (b) If $a|2b$ then $a|b$.

   (c) If $7|4a$ then $7|a$.

   (d) If $p|ab$ then $p|a$ or $p|b$

   (e) The set of natural numbers is closed under subtraction.

   (f) The set of rationals numbers is closed under division.

   (g) The set of rationals numbers is closed under multiplication.

   (h) The contrapositive of a statement is true if and only if the statement is true.

   (i) The converse of a statement being true is a sufficient condition for the statement to be true.

7. Be able to give complete and precise mathematical definitions for the terms “even”, “odd’, “divides”, “congruent (mod n)”, “greatest common divisor”, “least common multiple”, “rational number”, “irrational number”. Be able to state the division algorithm.

8. Given the following proposition, what would you assume and what would you need to show if you were proving it directly: “Given integers a and b, if $4|(a^2 + b^2)$, then a and b are not both odd”? Repeat the question if you were proving it by contrapositive, and if you were proving it by contradiction.

9. Find a number n between 0 and 6 (inclusive) such that $n \equiv 4325 \times 5462 \pmod 9$.

10. What is the last digit of 56325^{13}?

11. Find a number between 0 and 12 (inclusive) that is an additive inverse of $3 \pmod{13}$. How about a multiplicative inverse of $3 \pmod{13}$?

12. On planet Zorglob, days have 17 hours. If it is currently 5 o’clock, what time will it be in 173402 hours? Explain.

13. You should practice by doing many proofs. There are lots of examples in the exercises of Chapters 4, 5 and 6. The odd ones all have worked solutions in the back of the textbook, which I encourage you to take advantage of.