Math 2001, Midterm 2 **Name:** ______________________________

Part I: Concepts and Skills, short answer

1. Fully negate each of the following statements.

   (a) $\forall n \in \mathbb{N}, \exists m \in \mathbb{N}, m < n$.

   (b) $(R \vee Q)$ only if $(P \wedge Q)$.

   (c) For all real numbers x and y, there is a rational number z such that $x < z < y$.

2. Write down a complete and precise mathematical definition of "divides"

3. Four children take turns feeding their pet chickens. Today is Ardie's turn, tomorrow is Billie's, then Cody, then Danny, then back to Ardie, etc. Whose turn will it be in 365 days? Briefly explain your reasoning.

4. Find an integer from 0 to 10 that is congruent to $92^{33} \pmod{11}$. (Your logic and reasoning should be clearly discernible from your work.)

5. For each of the following statements, determine if it is true or false statement. Circle "T" or "F". You do not need to show any work.

T / F a. If $a, b \in \mathbb{Z}$, with $b > 0$, then there exist integers q and r such that $a = qb + r$, and $0 \leq r < b$.

T / F b. If $a|bc$ then $a|b$ or $a|c$

T / F c. If a and b are integers and $3|a$ and $3|b$, then $\gcd(a, b) \geq 3$.

6. Say you were going to use the technique of proof by contradiction to prove the following statement: Given integers a and b, if $4|(a^2 + b^2)$, then a is even or b is even. Clearly state all the assumptions you would make to begin your proof. (You do **not** need to complete the proof.)

Part II: Proofs

7. Choose one of the following propositions, and prove it by using contrapositive proof. The first one is easier, and is not worth the full number of points.

   (a) (max 17 points) Suppose $x \in \mathbb{Z}$. If $x^3 - 1$ is even, then x is odd.

   (b) (max 20 points)If $4|(a^2 + b^2)$, then a and b are not both odd.

8. Choose one of the the following propositions and prove it. The first one is easier, and is not worth the full number of points:

   (a) (max 17 points) If n is an odd integer, then $n^2 \equiv 1 \pmod 4$.

   (b) (max 20 points) If n is an odd integer, then $n^2 \equiv 1 \pmod 8$.