Proposition: when $h>0$ the inequality $(1+h)^n > 1+nh$ holds for all positive integers $n \geq 2$

Proof style 1 (using the "P(n)" terminology)

Proof (induction).

Let $P(n)$ be the open sentence "$(1+h)^n > 1+nh$"

Step 1: when $n=2$,

$$(1+h)^n = (1+h)^2$$
$$= 1+2h+h^2$$
$$> 1+2h \quad (\text{since } h^2>0)$$
$$= 1+nh.$$

So we have shown $P(2)$ is true. The base case is shown.

Step 2: Suppose $k \in \mathbb{N}$, $k \geq 2$. Suppose $P(k)$ is true, that is, assume $(1+h)^k > 1+kh$. This is the inductive hypothesis. We need to show $P(k+1)$ is true. That is, we must show $(1+h)^{k+1} > 1+(k+1)h$.

Consider the left-hand side:

$$(1+h)^{k+1} = (1+h)^k(1+h)$$
$$> (1+kh)(1+h) \quad (\text{by the inductive hypothesis})$$
$$= 1+kh+h+kh^2$$
$$= 1+(k+1)h+kh^2$$
$$> 1+(k+1)h \quad (\text{since } kh^2>0)$$

We have shown $P(k+1)$ is true.

Step 3: Since $P(2)$ is true and $\forall k \in \mathbb{N}$, $k \geq 2$ $P(k) \Rightarrow P(k+1)$, by mathematical induction $(1+h)^n > 1+nh$ for all natural numbers n, $n \geq 2$.

Proof style 2:

Proof (induction):

Step 1: when $n=2$, we must show $(1+h)^2 > 1+2h$.

$$(1+h)^2 = 1+2h+h^2$$
$$> 1+2h \quad (\text{since } h^2>0)$$

The induction is anchored.

Step 2: Suppose the statement is true when $n=k$. In other words, assume the inductive hypothesis $(1+h)^k > 1+kh$.

Then
$$(1+h)^{k+1} = (1+h)^k(1+h)$$
$$> (1+kh)(1+h) \quad (\text{by the inductive hypothesis.})$$
$$= 1+kh+h+kh^2$$
$$= 1+(k+1)h+kh^2$$
$$> 1+(k+1)h. \quad (\text{since } kh^2>0)$$

We have shown the statement is true for $n=k+1$.

Step 3: By mathematical induction, since the statement is true for $n=2$, & ~~if~~ it is true for $n=k$ it is necessarily true for $n=k+1$, we have shown

$$(1+h)^n \geq 1+nh$$

for all natural numbers $n \geq 2$

proposition: If n is a positive integer, then $64 \mid (3^{2n+1} + 40n - 3)$.

Proof (by induction)

when $n = 1$, $3^{2n+1} + 40n - 3 = 3^3 + 40 - 3 = 27 + 40 - 3 = 64$.

So when $n=1$, $64 \mid 3^{2n+1} + 4n - 3$. The induction is anchored.

Now suppose $k \in \mathbb{N}$ and $64 \mid (3^{2k+1} + 40n - 3)$. By the definition of divides, we can choose $a \in \mathbb{N}$ such that $3^{2k+1} + 40n - 3 = 64a$. Now suppose $k \in \mathbb{N}$. we must show $64 \mid (3^{2(k+1)+1} + 40(k+1) - 3)$

$$3^{2(k+1)+1} + 40(k+1) - 3 = 3^{2k+3} + 40k + 40 - 3$$

$$= 3^2 \cdot 3^{2k+1} + 40k + 37$$

$$= 9 \cdot 3^{2k+1} + \underbrace{9 \cdot 40k - 9 \cdot 40k}\ \overbrace{-9 \cdot 3 + 9 \cdot 3} + 40k + 37$$

(by adding and subtracting the same number)

$$= 9 \cdot 3^{2k+1} + 9 \cdot 40k - 9 \cdot 3 - 9 \cdot 40k + 40k + 9 \cdot 3 + 37$$

$$= 9(3^{2k+1} + 40k - 3) - 320k + 64$$

$$= 9(64a) + 64(5k + 1)$$

$$= 64(9a + 5k + 1)$$

By the definition of divides, $64 \mid (3^{2(k+1)+1} + 40(k+1) - 3)$.

By induction, $64 \mid (3^{2n+1} - 40n - 3)$ for all natural numbers n.