Math 2001 - Group Homework 7

1. In your new groups, choose a leader, scribe and presenter for this week. Each person should be taking their fair share of each job throughout the semester.

2. Your group's work product will be the filled-in Report on Group Work, along with the solutions to Main Tasks 2 and 3 below. These can both be handwritten, but must be written neatly. Turn both of these in on the due date.

3. As before, for the rest of the meeting, the leader should record on the Report of Group Work any questions for the instructor (anything your group was not able to resolve together), along with a basic narrative description of events of the meeting. Help the leader by suggesting how to briefly summarize what transpired, and wait for the leader to finish. As before, the scribe is responsible for writing up the solutions to the assigned problems. The scribe must also share copies of the returned homework with the rest of the group.

4. **Main Task 1:** Review the daily homework, final-draft homework, reading, quizzes, returned group homework and in-class work covered since the last meeting. Everyone should take out old homeworks and last week's group homework, and go over solutions to each problem. Take turns, each person sharing their answers with the group. Ask questions of each other until everyone understands everyone else's answers and all questions and concerns have been resolved. If something is unresolved, put it on the list of questions for me.

5. **Main Task 2: Proof by induction:** Prove the following results, working together as a group:

   (a) Any amount of postage 14¢ or greater can be formed using just 8¢ and 3¢ stamps.

   (b) When $h > 0$, the inequality $(1+h)^n > 1 + nh$ holds for all positive integers $n \geq 2$

   (c) Show that if n is a positive integer, then $64|(3^{2n+1} + 40n - 3)$.

6. **Main Task 3:** Continue playing the game of Take-away, as we learned in class. You must spend at least 30 minutes on this task. Recall that the game is played by starting with a pile of n things. Two players take turns. On your turn you may take away either 1 or 2 of the things. The goal of the game is to be the one to take the last thing(s). During your meeting, work towards understanding the following:

   - When is it advantageous to go first?
   - When do you have a winning position?
   - What strategy should I use to win?
   - How do I formulate the above questions and answers mathematically?
   - How do I prove my conclusions?

   If your group should figure out all of the above to everyone's satisfaction, then begin considering the extensions below. Modify the game as follows: instead of taking either 1 or 2 things away, each player may take 1, 2, or 3 things away on his or her turn. Answer all of the above questions for this new game. If your group figures that out, then modify the game as follows, answering all of the above questions for the modified game: each player may take from 1 to r things away each turn. If your group figures that out, then modify the game as follows: Instead of one pile, there are two piles, starting with n_1 and n_2 objects. This time there is no limit on the number of things you can take away, but you may only take from one pile on each turn. The goal is still to take the last thing(s).

   Write down a summary of your conjectures and conclusions.

7. Before the meeting ends, help the group leader make sure that the Report on Group Work has been filled in and everyone has signed it. The scribe will take home the final solutions you have written together as a group, and typeset them (with latex), or type them, or write them up neatly and turn them in. I recommend that the scribe send the final copy to the groupmates to proofread before submitting them.