Functions, Part 3

- Definition: Suppose $f : A \mapsto B$ and $g : B \mapsto C$ are functions. The *composition* of f with g is a function $g \circ f : A \mapsto C$. For $x \in A$, $g \circ f(x) = g(f(x))$.
- Definition: Given a set A, the *identity function* on A is the function $i_A : A \mapsto A$ defined as $i_A(x) = x$ for every $x \in A$.
- Given a relation R from A to B, the *inverse relation of* R is the relation from B to A defined as $R^{-1} = \{(y, x) : (x, y) \in R\}$. In other words, the inverse of R is the relation R^{-1} obtained by interchanging the elements in every ordered pair in R.
- Theorem: The function $f : A \mapsto B$ is bijective if and only if the inverse relation f^{-1} is a function from B to A.
- Definition: If $f : A \mapsto B$ is bijective, then its *inverse* is the function $f^{-1} : B \mapsto A$.
- Key result regarding f and f^{-1}: The functons f and f^{-1} satisfy the equations $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$.

Example 1: Visual representation of composition

Example 2: Composition using formulas

Example 3: Inverse relations and inverse functions

Example 4: Visual representation of inverse functions

Example 5: Practice composition and inverses with ordered pairs

Suppose that $A = \{1, 2, 3\}$, $f : A \mapsto A$ and $g : A \mapsto A$, $f = \{(1, 3), (2, 1), (3, 2)\}$ and $g = \{(1, 2), (2, 3), (3, 2)\}$.

$f \circ g =$

$f \circ f \circ f =$

If f is bijective, then find f^{-1}

If g is bijective, then find g^{-1}

Example 6: Practice finding inverses with formulas and graphs

Suppose that both the domain and codomain of f are the positive reals, and say that for all x in the domain, $f(x) = x^2$. Is f invertible? If so, then find a formula for f^{-1}. Graph f and f^{-1} on the same graph.

Example 7: Practice finding inverses with formulas

Suppose that $f : \mathbb{R} - \{2\} \mapsto \mathbb{R} - \{1\}$ by $f(x) = \frac{x+1}{x-2}$. Find a formula for $f^{-1}(x)$. Then confirm algebraically that $f^{-1} \circ f = i_A$.