Functions, Part 1

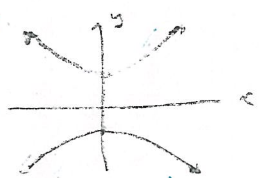
Definition: A *function* f from a set A into a set B (written $f : A \mapsto B$) is a relation $f \subseteq A \times B$ satisfying the property that for all $a \in A$ the relation f contains exactly one ordered pair of the form (a, b).. The statement $(a, b) \in f$ is also written $f(a) = b$. The set A is called the domain, and B is called the codomain.

You may be used to thinking of f as a map from $\mathbb{R}$ to $\mathbb{R}$ given by a formula, whose purpose is often to model some application. This is one use of a function, but the concept is actually much more general. The concept of a function will play a key role in every subsequent math class you take. It is one of tools we use to examine and compare mathematical structures.

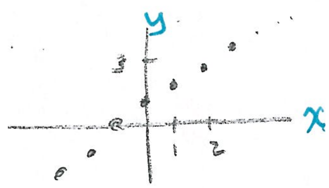
There are multiple ways to represent functions, including:

- Visually, with a graphs (this is familiar to you)
- With formulas (this is familiar to you)
- In a table (this should be familiar to you)
- With words (we do this often in precalculus and calculus)
- As a set of ordered pairs (this is probably new to you)
- As an arrow diagram (this is probably new to you)
- As a "machine" (not common, but I think it is sometimes useful)

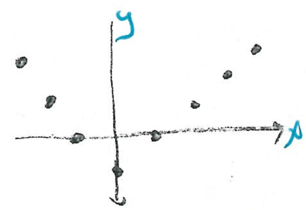
Cross out all of the examples below that are not functions:

~~1.~~


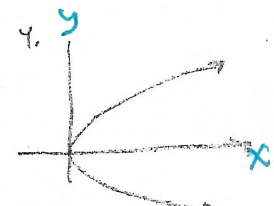
$y^2 - x^2 = 1$

2.


$f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = y$.
surjective, injective, bijective

3.


$f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = |x| - 1$
not injective, not surjective

4.


Does not give y as a function of x, but x is a function of y
$f: \mathbb{R} \to \mathbb{R}$ by $f(y) = y^2$
not injective, not surjective

~~5.~~ $f: \mathbb{N} \to \mathbb{N}$ by $f(x) = \sqrt{x}$

$f(2) = \sqrt{2}$, $\sqrt{2} \notin \mathbb{N}$

~~6.~~ $(x-1)^2 + (y+2)^2 = 4$

7.

a	b
1	1
2	1
3	2
4	2

Not injective. surjective if codomain $= \{1, 2\}$

~~8.~~

x	y
1	1
1	2
1	3

y is not a function of x.

9.

x	y
1	3
2	2
3	1

injective. surjective if codomain $= \{1, 2, 3\}$

~~10.~~ $R_1 = \{(1,1), (1,2), (2,5)\}$

11. $R_2 = \{(2,1), (3,1), (4,1)\}$

Not injective,
surjective if codomain $= \{1\}$

12. $R_3 = \{(1,2), (2,3), (3,1)\}$

injective.
surjective if codomain $= \{1,2,3\}$

13. $R_4 = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}, y = x^2\}$

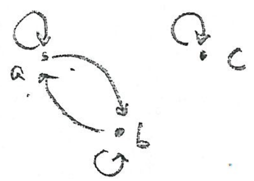
Not injective ($f(1) = f(-1)$)
not surjective ($f(x) = -1$ has no solutions)

14. $A = B =$ set of human beings
$f(a) =$ a's birth mother

not injective (siblings), not surjective (males)

~~15.~~ $A = B =$ set of human beings
$f(a) =$ a's friend

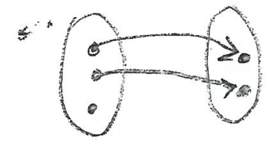
~~16.~~ $A = B = \{a, b, c\}$



17. $A = B = \{1, 2, 3\}$

injective
surjective
bijective

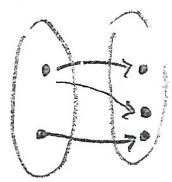
~~18.~~ $A \to B$



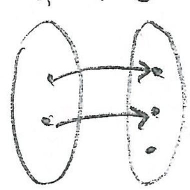
19. $A \to B$

not injective.
surjective

~~20.~~ $A \to B$

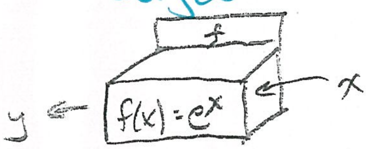


21. $A \to B$



injective,
not surjective

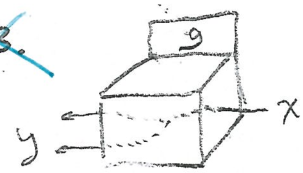
22.



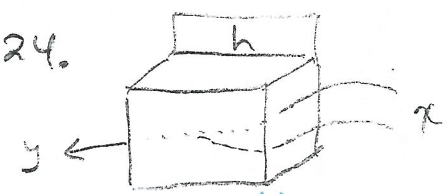
$f : \mathbb{R} \to \mathbb{R}$

injective, but not surjective. ($f(x) = -1$ has no solutions)

~~23.~~



24.



not injective, no way to determine surjectivity

Some more definitions:

1. A function $f : A \mapsto B$ is *surjective* (or "onto") if for every $b \in B$ there is an $a \in A$ with $f(a) = b$.
2. A function $f : A \mapsto B$ is *injective* (or "one-to-one") if for every $x, y \in A$, $x \neq y$ implies $f(x) \neq f(y)$.
3. A function $f : A \mapsto B$ is *bijective* (or "one-to-one and onto") if it is both surjective and bijective.

Return to the functions above, and determine which are surjective, which are injective and which are bijective.