Functions, Part 1

Definition: A *function* f from a set A into a set B (written $f : A \mapsto B$) is a relation $f \subseteq A \times B$ satisfying the property that for all $a \in A$ the relation f contains exactly one ordered pair of the form (a, b).. The statement $(a, b) \in f$ is also written $f(a) = b$. The set A is called the domain, and B is called the codomain.

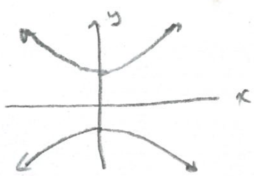
You may be used to thinking of f as a map from $\mathbb{R}$ to $\mathbb{R}$ given by a formula, whose purpose is often to model some application. This is one use of a function, but the concept is actually much more general. The concept of a function will play a key role in every subsequent math class you take. It is one of tools we use to examine and compare mathematical structures.

There are multiple ways to represent functions, including:

- Visually, with a graphs (this is familiar to you)
- With formulas (this is familiar to you)
- In a table (this should be familiar to you)
- With words (we do this often in precalculus and calculus)
- As a set of ordered pairs (this is probably new to you)
- As an arrow diagram (this is probably new to you)
- As a "machine" (not common, but I think it is sometimes useful)

Cross out all of the examples below that are not functions:

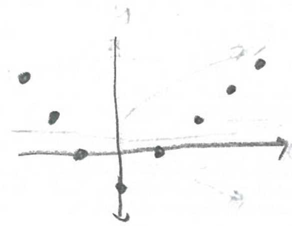
1.



$y^2 - x^2 = 1$

2.

3.



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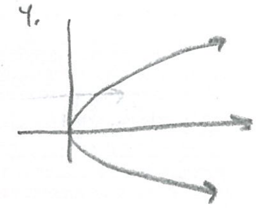


5. $f: \mathbb{N} \to \mathbb{N}$ by $f(x) = \sqrt{x}$

6. $(x-1)^2 + (y+2)^2 = 4$

7.

a	b
1	1
2	1
3	2
4	2

8.

x	y
1	1
1	2
1	3

9.

x	y
1	3
2	2
3	1

10. $R_1 = \{(1,1), (1,2), (2,5)\}$

11. $R_2 = \{(2,1), (3,1), (4,1)\}$

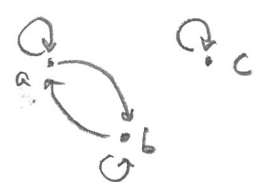
12. $R_3 = \{(1,2), (2,3), (3,1)\}$

13. $R_4 = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}, y = x^2\}$

14. A = B = set of human beings
f(a) = a's birth mother

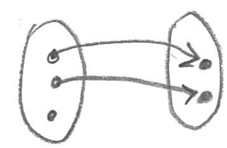
15. A = B = set of human beings
f(a) = a's friend

16 $A = B = \{a, b, c\}$

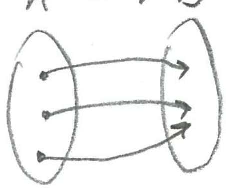


17. $A = B = \{1, 2, 3\}$

18. $A \to B$

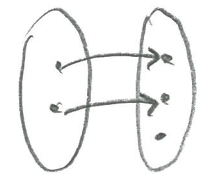


19. $A \to B$

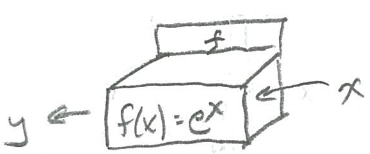


20. $A \to B$

21. $A \to B$

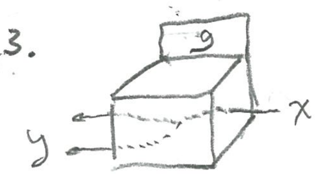


22.

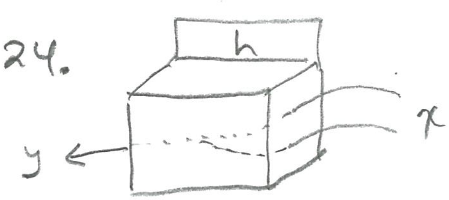


$f : \mathbb{R} \to \mathbb{R}$

23.



24.



Some more definitions:

1. A function $f : A \mapsto B$ is *surjective* (or "onto") if for every $b \in B$ there is an $a \in A$ with $f(a) = b$.
2. A function $f : A \mapsto B$ is *injective* (or "one-to-one") if for every $x, y \in A$, $x \neq y$ implies $f(x) \neq f(y)$.
3. A function $f : A \mapsto B$ is *bijective* (or "one-to-one and onto") if it is both surjective and bijective.

Return to the functions above, and determine which are surjective, which are injective and which are bijective.