Counting Worksheet 2 (order might not matter)

1. On a 12-person basketball team, how many ways are there to create a 5-person group of starters?

no replacement, order doesnt matter. Choose 5 from a set of 12

$$\binom{12}{5} = \frac{12!}{7!5!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{12\cdot 10}{120}\cdot 11\cdot 9\cdot 8 = 11\cdot 9\cdot 8 = 792$$

2. This problem concerns the Colorado lottery.

   (a) In the Colorado lottery (lotto), there are 42 ping-pong balls, numbered from 1 to 42. In a random drawing, 6 of the balls are chosen. How many possible outcomes are there?

   order doesn't matter. no replacement

   $\binom{42}{6} = 5{,}245{,}786$ (used a calculator)

   (b) If you choose your numbers randomly each time, and play 100 times in a year, what are your chances of winning the lotto three times?

   Chance of winning each time: $p = \frac{1}{\binom{42}{6}}$. Chance of losing $= q = 1-p$.

   chance of winning 3 specific times, losing the rest: $p^3 q^{97}$

   # of ways of picking the 3 specific times: $\binom{100}{3}$

   Chance of winning exactly 3 times: $\binom{100}{3} p^3 q^{97}$

   (c) Now suppose you choose the same numbers each time. How does this affect your chances?

   Not at all. The random drawing is not influenced by my choices; probabilities are unchanged.

3. A set X has 35 four-element subsets. What is $|X|$?

If $|X| = n$, we are told $\binom{n}{4} = 35$. By trial and error,

$$\binom{7}{4} = \frac{7!}{3!\cdot 4!} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$$

So $|X| = 7$

4. There are 10 people standing in line, including Joe and Jane. In how many of the possible arrangements of people in the line is Jane ahead of Joe?

It seems like it should be half of the $10!$ orderings, so $\frac{10!}{2}$ possible arrangements. another solution would be to choose the 2 spots for Jane & Joe. They can be placed in only one order. The remaining 8 people can be placed $8!$ ways. So $\binom{10}{2}\cdot 8! = \frac{10!}{2!\cdot 8!}\cdot 8! = \frac{10!}{2}$.

5. How many 5-digit postal-codes have exactly 3 zeroes?

choose the 3 positions for the zeroes. Each of the 2 remaining spots has 9 possibilities.

$$\binom{5}{3}\cdot 9^2 = \frac{5!}{3!2!}\cdot 9^2 = 10\cdot 9^2 = 810$$

6. Use the binomial theorem to expand $(x - 2y)^5$.

$x^5 y^0 + 5 \cdot x^4(-2y) + 10 \cdot x^3 \cdot (-2y)^2 - 10x^2(-2y)^3 + 5x(-2y)^4 + 1 \cdot x^0(-2y)^5$

$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 10xy^4 - 32y^5$

7. What is the coefficient of x^7y^3 in the expansion of $(x + y)^10$?

should say $(x+y)^{10}$.

x^7y^3 term: $\binom{10}{7} \cdot x^7 y^3 = \frac{10!}{7!3!} x^7 y^3$. Coefficient is $\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

8. What is the coefficient of x^6 in the expansion of $(x + 1)^9$?

coefficient is $\binom{9}{6} = \frac{9!}{6! \cdot 3!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84$

9. What is the coefficient of x^6 in the expansion of $(x - 3)^9$?

the term is $\binom{9}{6}x^6(-3)^3$, so the coefficient is

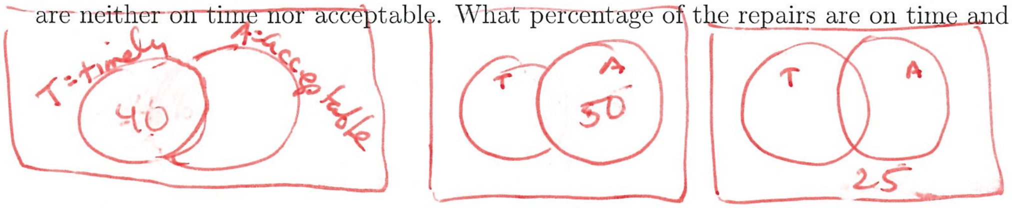
$\binom{9}{6}(-3)^3 = -27 \cdot 84 = -2268$

10. Find a formula for the sum of the numbers in row n of Pascal's triangle and explain why your formula is correct.

If I expand $(1+1)^n$, I get:

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$, the sum of row n of pascal's triangle. So the sum is $(1+1)^n = 2^n$

11. At a car repair shop, 40% of the repairs are on time, 50% of the repairs are acceptable, and 25% of the repairs are neither on time nor acceptable. What percentage of the repairs are on time and acceptable? Imagine there are 100 cars



Timely or Acceptable $= .75$

$75 = |T \cup A| = |T| + |A| - |T \cap A| = 50 + 40 - |T \cap A|$. $\Rightarrow |T \cap A| = 15$

15% are neither.

12. How many integers from 1 to 1000 (inclusive) are a multiple of 2 or a multiple of 5?

Let $T = \{2n \mid n \in \mathbb{N}, n \leq 500\}$; $|T| = 500$

$F = \{5n \mid n \in \mathbb{N}, n \leq 200\}$; $|F| = 200$

$T \cap F = \{10n \mid n \in \mathbb{N}, n \leq 100\}$: $|T \cap F| = 100$

$|T \cup F| = |T| + |F| - |T \cap F| = 500 + 200 - 100 = \boxed{600}$

13. We have 7 balls, each of a different color, that we are placing in 3 different boxes, each of a different size. How many ways are there to do this so that none of the boxes are empty?

If there is no restriction on emptiness, there are 3^7 possibilities.

Let B = set of ways the big box is empty, M = set of ways medium box is empty, S = set of ways the big box is empty. I want $3^7 - |B \cup M \cup S|$

$|B| = |M| = |S| = 2^7$. $|B \cap M| = |M \cap S| = |B \cap S| = 1$. $|B \cap M \cap S| = 0$.

$|B \cup M \cup S| = |B| + |M| + |S| - |B \cap M| - |B \cap S| - |M \cap S| + |B \cap M \cap S| = 3 \cdot 2^7 - 3 + 0$

Total $= 3^7 - (3 \cdot 2^7 - 3) = 1806$