SOLUTIONS

Math 2001

Practice with Conditional and Biconditional Statements

1. Give the truth table for $P \Rightarrow Q$.

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

2. Fill in the blanks below so that all of the statements below are equivalent to the conditional statement $P \Rightarrow Q$.

- If P then Q.
- Q, if P.
- Q, whenever P.
- P only if Q.
- P is a sufficient condition for Q.
- Q is a necessary condition for P.
- Q, provided P.

3. Suppose that X is a mome rath (Q) whenever X is a borogove (P). Which of the following is true? $P \Rightarrow Q$

NO ~~(a)~~ If X is a mome rath, then X is a borogove. $Q \Rightarrow P$

NO ~~(b)~~ X is a mome rath only if X is a borogove. $Q \Rightarrow P$

NO ~~(c)~~ Being a mome rath is a sufficient condition for being a borogove. $Q \Rightarrow P$

YES (d) Being a mome rath is a necessary condition for being a borogove. $P \Rightarrow Q$

NO ~~(e)~~ To be a mome rath, it is necessary to be a borogove. $Q \Rightarrow P$

4. Rephrase the following statement into "If-then" form:
"An integer is prime only if 2 does not divide it".
Is the statement true? Is the converse true?

If an integer is prime, then 2 does not divide it

False, because 2 itself is prime, yet 2 divides it.

5. Give the truth table for $P \Leftrightarrow Q$.

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

6. Write the biconditional statement $P \Leftrightarrow Q$ in words, in three distinct ways.

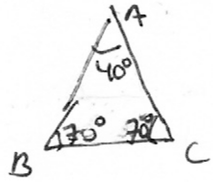
- P if and only if Q
- P is necessary and sufficient for Q
- If P then Q, and conversely

7. Rephrase the biconditional "The triangle ΔABC is isosceles if and only if $\angle A \cong \angle B$" in another way. Is the biconditional statement true or false? Explain.

"If the triangle ΔABC is isosceles, then $\angle A \cong \angle B$, and if $\angle A \cong \angle B$ then ΔABC is isosceles."

FALSE.

Although $\angle A \cong \angle B \Rightarrow \Delta ABC$ is isosceles is true (just by the definition of isosceles), the converse is not. That is, "If ΔABC is isosceles, then $\angle A \cong \angle B$" is false. As a counter-example, consider this example:



This is an isosceles triangle, but $\angle A \ncong \angle B$.