Chapter 1 and 2 Review problems

1. (a) If a triangle is equilateral, then it is isosceles (T)

(b) If your cell phone functions properly, then it is charged (T)

(c) If an animal is a mammal, then it is a bear (F)

(d) If a number is odd, then it is prime (F)

2. "Scaramouche only if fandango"

(a) not equivalent

(b) equivalent

(c) not equivalent

(d) not equivalent

(e) equivalent

(f) equivalent

3. (a) TRUE, choose $y = x+1$

(b) False, if $x = 1$, there are no natural numbers less than $x$

(c) TRUE. use $n = 0$.

(d) False. $n+m = n \Rightarrow m = 0$. But my statement is supposed to hold for all $m$.

(e) True. Given $n$, choose $m = -n$.

(f) False. $m$ needs to depend on $n$

(g) True. $x \notin \overline{A \cup B} \iff \neg(x \in A \text{ or } x \in B) \iff x \notin A \text{ and } x \notin B \iff x \in \overline{A} \cap \overline{B}$

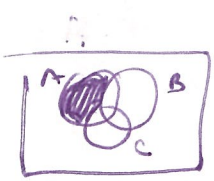
4. (a) Always. There is an $m \in S$ s.t. $P(n,m)$ is true for all $n$. The second statement allows $m$ to depend on $n$, which is less restrictive.

(b) Sometimes

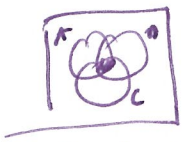
5. $\forall n \in \mathbb{N}, \exists q \in \mathbb{Q}, n = \sqrt[4]{q}$.

True. Choose $q = n^4$.

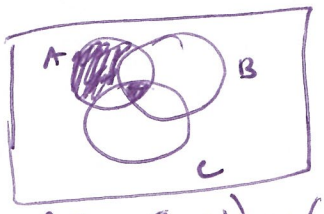
6.




$A - (B \cup C)$




$A \cap B \cap C$




$(A - (B \cup C)) \cup (A \cap B \cap C)$

7. True, we discussed this in class. Use a truth table to prove. Make sure it makes sense intuitively, too

8. $\{6, 18, 54, 162, \ldots\} = \{2 \cdot 3^n : n \in \mathbb{N}\}$.

9. a) yes, $\emptyset$

b) not valid notation

c) yes, $\mathbb{Z}$

d) yes, $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

e) not valid notation

f) Not valid notation, n undefined

10. $|N \cap M| = 2$ ; $|N \times (N \cap M)| = 3 \cdot 2 = 6$

$|\mathcal{P}(N \times (N \cap M))| = 2^6$

11. a) False. (consider elements that are in A and and not in C)

b) False, the parentheses are different in the two different sets

(c) False

(d) True

(e) True

(f) True

(g) False

(h) True

(i) False

(j) True

(k) False

(l) True

(m) True

(n) True

(o) False

12. $P \Rightarrow Q = Q \vee \sim P$

13. $P \Rightarrow Q = \sim\ \sim(Q \vee \sim P) = \sim(\sim Q \wedge P)$

14. $P \Leftrightarrow Q = (P \Rightarrow Q) \wedge (Q \Rightarrow P) = (Q \vee \sim P) \wedge (P \vee \sim Q)$

15. a) P: there is a set containing all sets
Q: Set theory is inconsistent.

$P \Rightarrow Q$

b) P: She doesn't take him literally
Q: She will agree that there is an elephant in the room

$P \Rightarrow Q$

c) P: a subset X of a topological space T is compact
Q: Every open cover of X contains a finite subcover.

$P \Leftrightarrow Q$