

Chapter 1 and 2 review problems

- Rewrite each of the following as an equivalent statement of the form if _____, then _____:
 - A triangle is isosceles whenever it is equilateral.
 - For your cell phone to function properly, it is necessary for it to be charged.
 - An animal is a mammal only if it is a bear.
 - For a number to be prime, it is sufficient that it be odd.

(Notice that the task is much more difficult when the statement itself is false. Refer to page 43 of the text for a list of grammatical structures that mean the same thing as if _____, then _____.)
- Circle any of the following statements that are equivalent to “Scaramouche only if fandango.”
 - If fandango then scaramouche.
 - Fandango whenever scaramouche.
 - Fandango if and only if scaramouche.
 - Scaramouche is necessary for fandango.
 - Scaramouche is sufficient for fandango.
 - Fandango, provided scaramouche.
- Are each of the following statements true or false? Explain.
 - $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y > x$.
 - $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y < x$.
 - $\exists n \in \mathbb{Z}, \forall m \in \mathbb{Z}, n + m = m$
 - $\exists n \in \mathbb{Z}, \forall m \in \mathbb{Z}, n + m = n$.
 - $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, n + m = 0$
 - $\exists n \in \mathbb{Z}, \forall m \in \mathbb{Z}, n + m = 0$
 - Say A and B are sets with the same universal set U . Then $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
- Answer the questions below. Explain your reasoning.
 - Suppose that $\exists m \in S, \forall n \in S, P(n, m)$ is true. Is the statement $\forall n \in S, \exists m \in S, P(n, m)$ true sometimes, always, or never?
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- Translate the following sentence into symbolic logic: “For every natural number n , $n = \sqrt[4]{q}$ for some rational number q ”. Is it true or false?
- Draw a Venn diagram for $(A - (B \cup C)) \cup (A \cap B \cap C)$.
- Determine whether or not $\sim (P \implies Q)$ is logically equivalent to $P \wedge \sim Q$. If so, explain intuitively why this makes sense.
- Write the set $\{6, 18, 54, 162, \dots\}$ in set-builder notation.
- For each of the following, determine if it is proper notation for a set. If so, then write the set by listing its elements between braces. If not, explain why not.
 - $\{x : x = 1 \wedge x = 0\}$
 - $\{x = 0\}$
 - $\{q \in \mathbb{Q} : q \in \mathbb{Z}\}$
 - $\{x \in \mathbb{Z} : |\frac{x}{2} - 1| < 3\}$
 - $\{3x + 1 < 5 : x \in \mathbb{Z}\}$
 - $\{5x^n : x \in \mathbb{N} \text{ and } |x| < 2\}$

10. If $N = \{1, 2, 3\}$, $M = \{1, 3, 5\}$, then what is $|\mathcal{P}(N \times (N \cap M))|$?
11. Determine whether the following claims are true for all sets A, B , and C , with universal set U . If true, explain why. If false, give a specific example that shows that the claim does not hold.
- (a) $(A \cup B) \cap C = A \cup (B \cap C)$
 - (b) $(A \times B) \times C = A \times (B \times C)$
 - (c) $(A - B) - C = A - (B - C)$
 - (d) $(A \cup B) - C = (A - C) \cup (B - C)$
 - (e) $U - \overline{A} = A$
 - (f) If $A \cap C = \emptyset$, then $(A \cap B) - C = A \cap (B - C)$.
 - (g) If $A - B = \emptyset$, then $A = B$.
 - (h) $A \in \mathcal{P}(A)$
 - (i) $A \subseteq \mathcal{P}(A)$
 - (j) $A \not\subseteq \mathcal{P}(A)$
 - (k) $|\mathcal{P}(A - \{\emptyset\})| = 2^{|A|}$
 - (l) $|\mathcal{P}(A) - \{\emptyset\}| = 2^{|A|} - 1$
 - (m) $B \times \emptyset \subseteq A \times C$
 - (n) $\mathbb{Z} \times (\mathbb{R} - \mathbb{Z}) \subseteq \mathbb{R} \times \mathbb{R}$
 - (o) $\mathbb{Z} \times \mathbb{Z} = \{(a, b) \in \mathbb{N} \times \mathbb{N}\} \cup \{(-a, -b) : a, b \in \mathbb{N} \cup \{0\}\}$
12. Write the statement $P \Rightarrow Q$ using only the symbols $\sim, \vee, P$, and Q (and parentheses, if needed).
13. Write the statement $P \Rightarrow Q$ using only the symbols $\sim, \wedge, P$, and Q (and parentheses, if needed).
14. Write the statement $P \Leftrightarrow Q$ using only the symbols $\sim, \vee, P$, and Q (and parentheses, if needed).
15. Convert the following sentences into formal logic (decide how to define P, Q , etc.):
- (a) There is a set containing all sets only if set theory is inconsistent.
 - (b) She will agree that there is an elephant in the room provided she doesn't take him literally.
 - (c) A subset X of a topological space T is compact if and only if every open cover of X contains a finite subcover.