Algebra Lemmas

You may use the following axioms, lemmas, and properties in your proofs.

Lemma 1: $\mathbb{Z}$ is closed under addition.
Lemma 2: $\mathbb{Z}$ is closed under multiplication.
Reflexive Axiom: For $a \in \mathbb{Z}$, $a = a$.
Symmetric Axiom: For $a, b \in \mathbb{Z}$, if $a = b$, then $b = a$.
Transitive Axiom: For $a, b, c \in \mathbb{Z}$, if $a = b$ and $b = c$, then $a = c$.
Additive Axiom: For $a, b, c \in \mathbb{Z}$, if $a = b$, then $a + c = b + c$.
Additive property of inequality: For $a, b, c \in \mathbb{Z}$, if $a < b$, then $a + c < b + c$.
Multiplicative property of inequality: For $a, b, c \in \mathbb{Z}$, if $a < b$, and $c > 0$, then $ac < bc$.
Additive identity: For all $a \in \mathbb{Z}$, $a + 0 = a$.
Multiplicative identity: For all $a \in \mathbb{Z}$, $a \cdot 1 = a$.
Additive inverses: For all $a \in \mathbb{Z}$, there exists an integer $-a$ such that $a + (-a) = 0$.
Multiplicative Axiom: For $a, b, c \in \mathbb{Z}$, if $a = b$, then $ac = bc$.
Zero Axiom of Multiplication: For $a, b \in \mathbb{Z}$, if $ab = 0$, then $a = 0$ or $b = 0$.
Cancellation law: For $a, b, c \in \mathbb{Z}$, suppose $c \neq 0$. If $ac = bc$, then $a = b$.
Cancellation of negatives: $-1 \cdot -1 = 1$.
Commutative Law of Addition: For $a, b \in \mathbb{Z}$, $a + b = b + a$.
Commutative Law of Multiplication: For $a, b \in \mathbb{Z}$, $ab = ba$.
Associative Law of Addition: For $a, b, c \in \mathbb{Z}$, $(a + b) + c = a + (b + c)$.
Associative Law of Multiplication: For $a, b, c \in \mathbb{Z}$, $(ab)c = a(bc)$.
Distributive Law: For $a, b, c \in \mathbb{Z}$, $a(b + c) = ab + ac$.