MATH 2001, Discrete Math

Reading definitions carefully to find errors and ambiguities

February 2, 2017

If A and B are sets, the union of A and B (written $A \cup B$) is the set $A \cup B = \{x : x \in A \text{ and } x \in B\}$.

Oops, it should be “or”, not “and”. This is actually the definition of intersection, not union.

If A and B are sets, the union of A and B (written $A \cup B$) is $\{x : x \in A, x \in B\}$.

Should I interpret the comma as an "and" or as an "or"? In the textbook, commas in this position are treated as "and". But then this is the definition for intersection, not union.

Now we have examples of definitions written in words rather than symbols:

If A and B are sets, A union B is a set containing all elements in both A and B.

The placement of the word "both" creates an ambiguity here. Do I apply it to the two sets A and B, or to the individual elements? This looks like $A \cap B$ to me. Though I can see how someone could interpret it oppositely. This is an ambiguous definition.

If A and B are sets, A union B is a set containing all of the elements of A and all of the elements of B.

Slight variation on the previous example, and still not okay. According to this definition, it looks like $\{1\} \cup \{2\}$ could be $\{1, 2, 3\}$.

If A and B are sets, the <u>union</u> of A and B written $A \cup B$ is the set of all elements in A or in B.

Hmmm..., so I can pick whether it contains all the elements of A or all of the elements of B? So $A \cup B = A$ or $A \cup B = B$?

$$A \cup B = \{x : x \in A - B \text{ or } x \in B - A \text{ or } x \in A \cap B\}.$$

Please define all variables before you use them. Also this definition of $A \cup B$ as written relies the definition of set difference and set intersection. This is a really interesting fact, that would work well as a theorem.