Advice on Studying in Early Graduate School

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In graduate school, and increasingly as time goes by, you will be entirely responsible for your own learning. You can no longer expect your teachers to dictate study methods. They are resources, as are your texts and peers. They will devote their energy to giving you the tools, advice and raw materials needed, but you decide how to make them your own. They will, in fact, eventually disappear, and all you will have left is books.

This is not because they are unhelpful, but because there is no other effective way to learn mathematics than to make it your own in your own way. Therefore, you should choose study methods that match your style. Even so, there are several guiding principles to keep in mind.

1. **Repeated passes.** This is the simplest observation, but is sometimes forgotten. You do *not* achieve your best possible understanding the first time you learn something. In fact, you never do. It's a never-ending process, in which your understanding is always improving. This is why you read your textbook before and after class, read other books, take the same class repeated times, do homework, write notes, etc.

2. **Self questions.** Reading (or listening to) mathematics is an active endeavour. That means after each new definition, theorem statement, remark, etc., you ask yourself questions. For example, "what counterexample arises if I alter the hypothesis in this small way?", "how does this example I'm familiar with fit into this new setting?" etc. If you are not doing this, you may as well just be sounding it out in romanian.

3. **Underlying idea.** All too often (but not always!), written presentations of mathematics fail to explain the 'underlying idea'. For example, an introduction to groups may begin with the definition of a group instead of any suggestion as to why that definition is natural. A proof

may consist only of eleven technical steps. It is often your task to discover the underlying ideas (there are often many) by pondering not just the technicalities but the big picture. This apparent lack in many written presentations is not always a disadvantage: you will own the ideas you have worked hard to discover or construct yourself.

4. **Do not confuse familiarity with understanding.** It is easy to become familiar with facts. It is even easier to confuse this feeling with understanding. Strive for real understanding! Don't apply facts you can't explain. Don't trust theorems you can't work examples to.

5. **Focus your questions.** If you are confused, try to pinpoint with great precision exactly where your confusion lies. Be ruthless in this search. It can be helpful to start an email, math.stackexchange or mathoverflow post, where you are writing your question to a real person. In trying to explain yourself in written form, you will often narrow down the misunderstanding to a point where you can finally eliminate it. Most of my emails/posts begun this way are never needed. Avoid questions that put the burden on the answerer, like "I don't get this proof." or "I'm working this problem and I get to this step and then I don't know what to do next." Instead, ask yourself why you did the first few steps, what your final goal and overall strategy is, what type of step is needed next, and why the ideas you had for that step aren't appropriate and how they should be adapted to be helpful, why you can't adapt them that way, what information you are seeking at this point and what definitions or theorems may be relevant, etc. etc. If, after you've answered all that, you still have a question, it will be a much better informed question. Then you can go ask it.

6. **Doubt yourself.** One of the biggest challenges is to be able to identify what you do and don't know. Always doubt yourself, asking, "Do I *really* understand this?" The answer is never "yes." Don't think you understand something because you can quote it or answer problems correctly. Learn to listen to that niggling doubt in the back of your mind that says, 'something worries me...' and poke and prod that 'something' until it turns into a well-defined question. The answer to "Do I really understand?" can be "fairly well," but that is only achieved when you can teach the material from start to finish using only your *understanding* (not memory!). You can always improve your understanding.

7. **Trust yourself.** On the other hand, you shouldn't blindly do a bazillion exercises; you need to decide when you have achieved the level of understanding you can be satisfied with for now and move on to the next thing. (Better understanding usually comes from new challenges and novel contexts, not base repetition.) Once you have gotten in the habit of doubting yourself sufficiently, then you can learn to trust yourself and feel confident in your knowledge. You will know you understand something because you really tested yourself.

8. **What you build, you know.** If you can reconstruct a definition or proof from the underlying idea alone, you know you have made good progress toward understanding it. This is the best way to test yourself.

9. **Teaching.** You will always understand something better through teaching it, whether that takes the form of writing course notes (i.e. not copying board notes, but writing a book) for yourself, collaborating with other students on homeworks, or presenting in the graduate student seminar etc.

10. **Never memorize.** It is only worth knowing things you can construct, explain, or at least put naturally in context by yourself. And if you can do this then it means you won't need to memorize it. I don't mean you can't ever state Fermat's Last Theorem without knowing the proof, but you shouldn't state it unless it fits in a natural framework for you – there's no point in memorizing the equation $x^n + y^n = z^n$, or any other equation, as a sequence of letters. (Have you ever made a mnemonic for the characters in your favourite TV show? If you engage with the material, the important things will enter your brain naturally.)

11. **Talk to people.** Collaborate on assignments. Just find people, and do it. And when you need a break, just chat. Muse. Daydream. Wonder. Whatever. It can be invaluable.

12. **Make it fun.**