Sets Quiz (Katherine E. Stange, Math 2001, Spring 2023, CU Boulder)

Name:

Correct answers without justification will receive full credit (unless justification is required by the question). Incorrect answers with explanation can receive partial credit. If the questions are unclear, please ask during the test and I will clarify.

1. Give an example of a set A such that $|A| = 3$ and $1 \in A$.

   It should have 3 elements, one of which is 1.

   eg. $\{1,2,3\}$ or $\{7, 1, \text{socks}\}$

2. Compute the power set of $\{1,2,3\}$.

   $$\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

   Common error: putting $\{\emptyset\}$ instead of $\emptyset$; $\{\emptyset\}$ is not a subset of $\{1,2,3\}$. The power set is the set of all subsets.

3. If $|X| = n$, then compute $|\mathcal{P}(X)|$.

   2^n

4. Consider the sets $A = \{1,2,3\}$ and $B = \{-1,0,1\}$.

   (a) Compute $A \cup B$.

   $\{-1, 0, 1, 2, 3\}$

   (b) Compute $A \cap B$.

   $\{1\}$

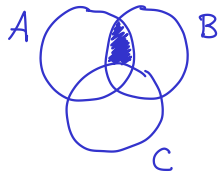
   (c) Compute $A - B$.

   $\{2, 3\}$

   (d) Compute the complement of A in the universe $\{1,2,3,4,5\}$.

   $\{4, 5\}$

5. Let A, B and C be arbitrary sets. Draw a Venn diagram illustrating $(A \cap B) - C$.



6. Give set builder notation for the integers between 1 and 10 (inclusive). Use symbols exclusively (not english words).

$\{x \in [1,10] : x \in \mathbb{Z}\}$
or $\{x \in \mathbb{Z} : 1 \le x \le 10\}$
or $\{x \in \mathbb{N} : 1 \le x \le 10\}$
or $\{x \in \mathbb{Z} : 0 < x < 11\}$

Common error: $\{x : 1 \le x \le 10\}$ (could be real numbers, not integers? doesn't clarify)

or $\{x : x \in [1,10], x \in \mathbb{Z}\}$
or even $\{x \le 10 : x \in \mathbb{Z}, 1 \le x\}$ etc.

7. Give an example of a set A such that $\{1\} \in A$ but $\{1\} \not\subseteq A$.

Should have $\{1\}$ as an element but not have 1 as an element.

eg. $\{\{1\}\}$ or $\{\{1\}, 3\}$ etc.

incorrect: $\{\{1\}, 1\}$ or $\{1\}$ (has 1 as an element).

8. Give set builder notation for the set of points in the Cartesian plane that lie on the line $y = 3x + 1$. Use symbols exclusively (not english words).

$\{(x,y) \in \mathbb{R} \times \mathbb{R} : y = 3x+1\}$
or $\{(x,y) \in \mathbb{R}^2 : y = 3x+1\}$
or $\{(x, 3x+1) : x \in \mathbb{R}\}$

9. Mark each of the following as TRUE, FALSE or NOT WELL-DEFINED (for example, if the set builder notation isn't valid – watch out for these!):

(a) $1 \in \{x \in \mathbb{R} : x\}$ not yes/no — TRUE, FALSE, NOT WELL-DEFINED

(b) $\{1\} \subseteq \{xy : x \in \mathbb{Z}, y \in \mathbb{Z}\}$
i.e. $1 \in \{xy : x \in \mathbb{Z}, y \in \mathbb{Z}\}$ — TRUE, FALSE, NOT WELL-DEFINED
$1 = 1 \cdot 1$ so $x = 1 = y$ creates the element 1

(c) $(-6, 4] \subseteq (-10, \infty)$
interval notation — TRUE, FALSE, NOT WELL-DEFINED

(d) $\{(x,y) : x \in \mathbb{R}, y \in \mathbb{Z}\} \subseteq \mathbb{R} \times \mathbb{R}$ — TRUE, FALSE, NOT WELL-DEFINED

(e) $\{x^2 : x \in \mathbb{R}\} = [0, \infty)$ — TRUE, FALSE, NOT WELL-DEFINED

(f) $\{1,2,3\} \in \{A \subseteq \mathbb{Z} : |A| = 2\}$
↑ does not have size 2 — TRUE, FALSE, NOT WELL-DEFINED

(g) $|\{0 < x < 3\}| = 2$
not set builder notation (no colon) ; also is it integers? real numbers? — TRUE, FALSE, NOT WELL-DEFINED

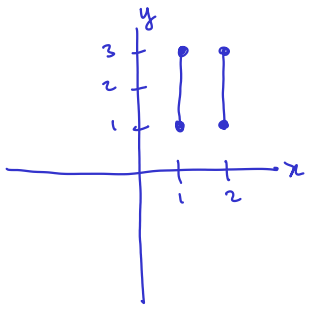
10. Let $|A| = n$ and $|B| = m$. What is the cardinality of $\mathscr{P}(A \times B)$?

$|A \times B| = |A| \times |B| = nm$

$|\mathscr{P}(A \times B)| = 2^{|A \times B|} = \boxed{2^{nm}}$

11. Is the following region of the plane a Cartesian product? If yes, gives sets A and B so that it is $A \times B$.

the region consists of 2 line segments



(YES.) NO

If yes, $A =$ $\{1, 2\}$

$B =$ $[1, 3]$

12. Compute the power set $\mathscr{P}(\emptyset)$.

This is the set of subsets of $\emptyset$. The only subset of $\emptyset$ is $\emptyset$.

$\mathscr{P}(\emptyset) = \{\emptyset\}$.

13. True or False: The empty set is an element of every set. If True, explain why. If False, give an example of a set for which the empty set is not an element.

False. For example, $\emptyset \notin \{1\}$

because $\emptyset$ is not one of the elements of $\{1\}$.

The only element of $\{1\}$ is 1.

14. Suppose $|A| = 5$. Can A be a Cartesian product? If not, explain why not. If so, give an example.

Yes, for example $A = \{1\} \times \{1, 2, 3, 4, 5\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5)\}$.