Sets

A **set** is a collection of **elements**.

- e.g. $\{a, b, c\}$ has 3 elements: a, b and c.
- may be **finite** or **infinite**
- **empty set** $\phi = \{\}$ has no elements
- elements can't repeat, e.g $\{1,1\}$ is not a set

Notation:

$\{1, 2, a, b\}$

elements separated by commas

braces indicate a set

Equality of sets: two sets are equal if they have the same elements.

- e.g. $\{1,2\} \neq \{1\}$ because 2 is in one set but not the other

  $\{1,2\} = \{2,1\}$ same elements (order doesn't matter!)

Cardinality: If X is a set, $|X|$ is the number of elements, called cardinality.

eg. $|\{1, a, 7\}| = 3$ $\quad |\mathbb{Z}| = \infty$

the set of integers

When a is an element of a set X, we write $a \in X$.
"is in"

eg. $1 \in \{1,2,3\}$ Otherwise, $a \notin X$.
"is not in"

$a \notin \{b, c\}$

Elements can be anything, even other sets:

eg. ① $S = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \emptyset, \{1\} \}$, $|S| = 3$, $\emptyset \in S$, $1 \notin S$

② $T = \{ \{1,2,3\} \}$, $|T| = 1$,

$\{1,2,3\} \in T$, $\emptyset \notin T$.

Set Builder Notation

to describe a set

$$\{\underbrace{\text{expression}}_{\text{type/form of element}} \overset{\text{"such that"}}{:} \underbrace{\overset{\text{property}}{\text{"rule"}}}_{\text{test it must pass to be included}}\}$$

examples:

$\{2n+1 : n \in \mathbb{Z}\}$ = things of the form $2n+1$ such that n is an integer

= odd integers

$= \{\ldots, -3, -1, 1, 3, 5, \ldots\}$

$\{n \in \mathbb{R} : n^2 = 5\}$ = real numbers such that they square to 5

$= \{\sqrt{5}, -\sqrt{5}\}$

$= \{x \in \mathbb{R} : x \in \{\sqrt{5}, -\sqrt{5}\}\}$

Special Sets:

$\mathbb{Z}$ = the set of integers $= \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{N}$ = the set of natural numbers $= \{1, 2, 3, \ldots\}$

= the set of positive integers $= \{n \in \mathbb{Z} : n > 0\}$

$\mathbb{R}$ = the set of real numbers $\quad \pi \in \mathbb{R}, \sqrt{2} \in \mathbb{R}$

$\mathbb{Q}$ = the set of rational numbers

$$= \left\{ \frac{p}{q} : p, q \in \mathbb{Z}, \ q \neq 0 \right\}$$

Inside $\mathbb{R}$, we often discuss intervals:

square bracket to include endpt

$[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

$(a,b) = \{x \in \mathbb{R} : a < x < b\}$

round bracket to exclude endpt

$(a,b] = \{x \in \mathbb{R} : a < x \leq b\}$

mixing allowed

$[a,\infty) = \{x \in \mathbb{R} : a \leq x\}$

∞ as "endpt" allowed, use round bracket

Def^n Suppose A, B are sets.
If every $x \in A$ satisfies $x \in B$ then $A \subseteq B$
"A is a subset of B".

Thm. If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

eg. $\mathbb{Z} \subseteq \mathbb{Q}$, $\mathbb{Q} \not\subseteq \mathbb{Z}$

every integer n can be expressed as $n = \frac{n}{1}$.

$\frac{1}{3} \in \mathbb{Q}$ but $\frac{1}{3} \notin \mathbb{Z}$.