

Math 2001: Setting up Proof by Contrapositive (Katherine Stange, CU Boulder)

For each theorem, set up the form of a proof by contrapositive. Do *not* write a full proof. That is, write the first sentence (or two) of the proof (that is, the assumptions), and the last sentence of the proof (that is, the conclusion). Do so in such a way as to demonstrate the structure of proof by contrapositive.

Theorem 1 (Example Theorem). *Let x be a real number. If x is irrational, then x has a non-periodic decimal expansion.*

1. First sentence(s): Suppose that x is a real number with a periodic decimal expansion.
2. Last sentence(s): Therefore x is rational.

Theorem 2. *Let n be an integer. If n^2 is a power of two, then n is a power of two.*

1. First sentence(s):
2. Last sentence(s):

Theorem 3. *For any integer n such that n^2 is odd, it must be that n is odd.*

1. First sentence(s):
2. Last sentence(s):

Theorem 4. *Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. If the derivative f' is identically zero, then f is a constant function.*

1. First sentence(s):
2. Last sentence(s):

Theorem 5. *Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function whose derivative is identically zero. Then f is a constant function.*

1. First sentence(s):
2. Last sentence(s):

Theorem 6. *Let n and m be negative integers. Then nm is a positive integer.*

1. First sentence(s):
2. Last sentence(s):