

1 Assignment

Prove that $\{6k + 1 : k \in \mathbb{Z}\} \subseteq \{3k + 1 : k \in \mathbb{Z}\}$.

Hint: We're getting kind of formal here, with lots of notation. Revisit the definition of 'subset' (the topic of the last class and today's reading) and determine what you need to do to prove that $A \subseteq B$ for sets A and B . Then try to apply that to this example.

2 A possible solution

Let us set notation for the two sets: $A = \{6k + 1 : k \in \mathbb{Z}\}$ and $B = \{3k + 1 : k \in \mathbb{Z}\}$.

To show that $A \subseteq B$, we must show that every element of A is an element of B . Suppose $x \in A$. Then $x = 6k + 1$ for some $k \in \mathbb{Z}$. But then $x = 3(2k) + 1$, which shows that $x \in B$.

3 A wordier solution

Let us set notation for the two sets: $A = \{6k + 1 : k \in \mathbb{Z}\}$ and $B = \{3k + 1 : k \in \mathbb{Z}\}$.

We wish to show that $A \subseteq B$. Let $x \in A$. We will show $x \in B$.

Since $x \in A$, $x = 6k + 1$ for some $k \in \mathbb{Z}$. Let us rewrite this expression:

$$x = 3(2k) + 1$$

Then, setting $\ell = 2k$, we see that $x = 3\ell + 1$ where ℓ is an integer. Therefore $x \in B$.

4 ChatGPT

One way to prove that a set A is a subset of a set B is to show that every element in A is also an element in B . To do this, we can use the definition of the sets in question, along with the properties of integers.

The set $\{6k + 1 : k \in \mathbb{Z}\}$ is defined as the set of all integers of the form $6k + 1$, where k is an integer. The set $\{3k + 1 : k \in \mathbb{Z}\}$ is defined as the set of all integers of the form $3k + 1$, where k is also an integer.

To show that $\{6k + 1 : k \in \mathbb{Z}\}$ is a subset of $\{3k + 1 : k \in \mathbb{Z}\}$, we can take an arbitrary element from the first set, say $x = 6k + 1$, and show that it can be expressed in the form $3k + 1$. We can do this by dividing both sides of the equation $x = 6k + 1$ by 3, which gives:

$$\begin{aligned} x/3 &= (6k + 1)/3 \\ x/3 &= 2k + 1/3 \end{aligned}$$

Since $x/3$ is an integer (because both x and 3 are integers), and $2k + 1/3$ is an integer, because k is an integer, we can say that $x = 3(2k) + 1$, which is in the form $3k + 1$.

Therefore, we can conclude that every element in $\{6k + 1 : k \in \mathbb{Z}\}$ can be expressed in the form $3k + 1$, which means that $\{6k + 1 : k \in \mathbb{Z}\}$ is a subset of $\{3k + 1 : k \in \mathbb{Z}\}$.

5 My comments on ChatGPT

If you just deleted the whole part from 'We can do this by dividing....' until '... and $2k + 1/3$ is an integer, because k is an integer', then the proof appears to have some logic. (The deleted portion is nonsense, not just extraneous.) What I find fascinating about this proof is that it is actually very similar to student errors I've seen for many years. After all, ChatGPT is an average of what you can find on the internet. This exact problem is on the internet, as are variations.

One big point: I dislike its use of variables. It says, ' $x = 3(2k) + 1$, which is in the form $3k + 1$.' but the k 's are the not the same here, so this will confuse the reader. Use a different variable.

ChatGPT would get a 3 out of 4 on writing and 2 out of 4 on logic.