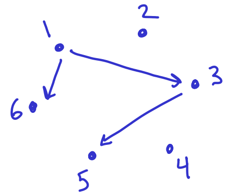
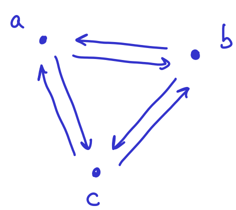
Relations Quiz (Katherine E. Stange, Math 2001, Spring 2023, CU Boulder)
Name: Solution Set

Correct answers without justification will receive full credit (unless justification is required by the question). Incorrect answers with explanation can receive partial credit. If the questions are unclear, please ask during the test and I will clarify.

1. Draw an arrow diagram of the relation $R = \{(1,3),(3,5),(1,6)\}$ on $A = \{1,2,3,4,5,6\}$.



2. Give the ordered pairs notation and the arrow diagram for the relation $\neq$ on the set $\{a,b,c\}$.



$\{(a,b),(b,a),(a,c),(c,a),(b,c),(c,b)\}$

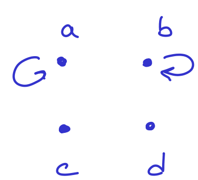
3. For each of the following relations, determine if it is reflexive, symmetric, transitive and/or equivalence.

   (a) The relation $<$ on $\mathbb{Z}$.
   - Reflexive? YES / NO — $1 \not< 1$
   - Symmetric? YES / NO — $1 < 2$ but $2 \not< 1$
   - Transitive? YES / NO
   - Equivalence? YES / NO

   (b) The relation $\neq$ on $\mathbb{Z}$.
   - Reflexive? YES / NO — $1 = 1$
   - Symmetric? YES / NO — if $a \neq b$ then $b \neq a$
   - Transitive? YES / NO — $1 \neq 2$ and $2 \neq 1$ but $1 = 1$
   - Equivalence? YES / NO

   (c) The relation $\emptyset$ on $\mathbb{Z}$. $\longrightarrow$ nothing is related to anything else
   - Reflexive? YES / NO — $1 \not R 1$
   - Symmetric? YES / NO $\}$ vacuously
   - Transitive? YES / NO
   - Equivalence? YES / NO

   (d) The relation $\{(a,a),(b,b)\}$ on $\{a,b,c,d\}$.
   - Reflexive? YES / NO — (c,c) missing
   - Symmetric? YES / NO
   - Transitive? YES / NO
   - Equivalence? YES / NO

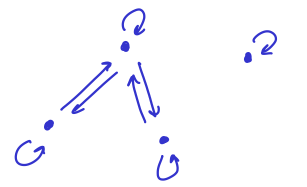


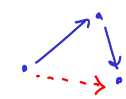
4. Give a relation on the set $\{a, b, c, d\}$ which is reflexive and symmetric but not transitive. (You can give an arrow diagram or a set of ordered pairs.)

must have self arrows

all arrows have back-arrows

some triangle is not complete



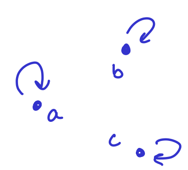


5. What is the equivalence class of 2 under the relation "has the same parity as" on the set $\{1, 2, 3, 4, 5, 6, 7\}$?

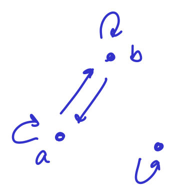
$\{2, 4, 6\}$ = all things with same parity as 2 (even)

6. Give an example of a partition of $\{a, b, c\}$ and the corresponding equivalence relation. (You can use an arrow diagram or set of ordered pairs for the equivalence relation.)

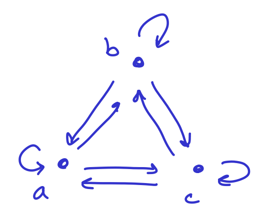
3 examples:



$\{a\}, \{b\}, \{c\}$



$\{a, b\}, \{c\}$



$\{a, b, c\}$

7. Compute $110003 + 14 \cdot 12 \pmod{11}$.

$\equiv 3 + 3 \cdot 1$

$\equiv 6$

8. Give at least 3 example elements of the equivalence class [3] modulo 8.

$-13, -5, 3, 11, 19, 27$ etc.

9. How many equivalence classes are there total modulo 7?

7 ; they are $[0], [1], [2], [3], [4], [5], [6]$

10. Let A be a finite set with $|A| = n$. Suppose that the equivalence classes of a relation R on A all have size exactly m. How many equivalence classes are there?

$$\frac{n}{m} = \frac{\#\text{ of elements}}{\#\text{ of elements per class}} = \#\text{ of classes}$$

11. How many possible relations are there on a set of size n?

2^{n^2} since a relation is a subset of the set $A \times A$ of ordered pairs, so $|\mathcal{P}(A \times A)| = 2^{|A \times A|} = 2^{|A|^2} = 2^{n^2}$

12. What is 1/2 modulo 7? Solve $2x \equiv 1 \pmod 7$ by inspection

$x \equiv 4$ (since $2 \cdot 4 \equiv 8 \equiv 1$)