

1 Quantifiers

For each english sentence, write it in symbols: **no english words!** You can use such things as quantifiers (\exists, \forall), boolean operators ($\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$), etc.

1. The set X has 3 elements.

$$|X| = 3$$

2. The number x is odd.

$$x \not\equiv 0 \pmod{2}$$

$$\sim E(x)$$

$$x \in \mathbb{Z} \wedge x \notin 2\mathbb{Z}$$

← not covered yet

$$\sim (x = 2k \text{ for some } k \in \mathbb{Z})$$

$$\forall k \in \mathbb{Z}, x \neq 2k$$

$$\sim (\exists k, x = 2k)$$

3. Every rational number is an integer.

$$\forall x \in \mathbb{Q}, x \in \mathbb{Z}.$$

4. Every integer is odd.

$$\forall x \in \mathbb{Z}, x \text{ is odd.}$$

5. If $x \in \mathbb{Z}$, then x is odd.

$$x \in \mathbb{Z} \Rightarrow x \text{ is odd.}$$

$$\forall x \in \mathbb{Z}, x \text{ is odd.}$$

6. If $x \in \mathbb{R}$, then $x \in \mathbb{Q}$.

$$\forall x \in \mathbb{R}, x \in \mathbb{Q}.$$

~~$E(n)$~~

$$E(n)$$

= "n is even"

$2\mathbb{Z}$ = set of even integers

7. There exists an odd integer.

$$\exists x \in \mathbb{Z}, x \text{ is odd.}$$

8. There exists a subset of the integers of cardinality 3.

$$\exists X \subseteq \mathbb{Z}, |X| = 3.$$

9. If X has 3 elements, then there exists a subset of X having 2 elements.

$$|X| = 3 \Rightarrow \exists Y \subseteq X, |Y| = 2.$$

10. For every integer, there's an additive inverse ^{in the integers} (something which adds to it to give 0).

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y = 0.$$

eg. negation: $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y \neq 0.$