Quantifiers and Negation

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1 Reading Quantifiers

In this section, consider the set $S = \{1, 2, \{2\}, 3, \{1, 2\}, \{4\}\}$.

For each statement, agree on what it means, and determine if it is true or false. If it is a false universal statement, give a counterexample. If it is a true existential statement, give the example that makes it true. To make this more fun, circling the letter under the correct column for each statement will spell something.

Statement	TRUE	FALSE	Example (if true existential)	Counterexample (if false universal)
$\exists x \in S, x = 2$	S	G		
$\exists x \in S, x = 4.$	A	E		
$\forall x \in S, x = 2.$	R	T		
$\forall x \in S, x \in \mathbb{Z}.$	T	S		
$\forall x \in S, x \subseteq \mathbb{Z}.$	C	A		
$\exists x \in \mathbb{Z}, x \in S.$	R	L		
$\exists x \in S, x \in \mathbb{Z}.$	E	A		
$\exists x \in S, \{x\} \in S.$	T	S		
$\forall x \in S, (x \in \mathbb{Z}) \vee (x \subseteq \mathbb{Z}).$	H	O		
$\forall x \in \mathbb{Z}, x \notin S.$	O	E		
$\forall x \in \mathbb{Z}, \{x\} \notin S.$	D	C		
$\forall x \in \mathbb{Z}, \{\{x\}\} \notin S.$	A	O		
$\forall x \in S, \{x\} \subseteq S.$	T	G		
$\forall x \in \mathbb{Z}, (1 \leq x \leq 4 \implies x \in S).$	H	S		
$\forall x \in \mathbb{Z}, (1 \leq x \leq 3 \implies x \in S).$	P	D		
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \in S).$	A	J		
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \subseteq S).$	S	W		

2 Quantifiers and Negation

Negate the following statements. Which is true, the original or the negation? If an existential statement is true, demonstrate it with an example. If a universal statement is false, demonstrate it with a counterexample.

Statement	T/F?	Example (if true existential)	Counterexample (if false universal)
$\forall x \in \mathbb{Z}, x > 3$			
Negation:			
$\forall x \in \mathbb{Z}, 0x = 0$			
Negation:			
$\exists x \in \mathbb{Z}, x$ is prime and negative			
Negation:			
$\exists x \in \mathbb{Z}, x = -x$			
Negation:			
$\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}, x = -y$			
Negation:			
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x = -y$			
Negation:			

3 Quantifiers and Proofs

Thanks to Elizabeth Gillaspy for some of these problems.

For each of the statements below, please do the following:

1. Decide if the statement is true or false.
2. If true, please prove it.
3. If the statement was false, please:
   (a) Modify the statement so that it's true.
   (b) Prove your revised statement.
4. Negate the original statement. Is the negated statement true or false?
5. If true, please prove it.
6. If the negated statement is false, please:
   (a) Modify the statement so that it's true.
   (b) Prove your revised statement.

The statements:

1. $\forall\ x \in \mathbb{N},\ \exists\ y \in \mathbb{N}$ such that $xy = 1$.
2. Let $A = \{1, 2, 3, 4\}$.
   (a) $\forall\ B \subseteq A,\ 3 \in B$
   (b) $\forall n \in \mathbb{N}$ such that $n \leq 4$, $\exists\ B \in 2^A, |B| = n$.
3. $\forall X, Y$ cards in the game SET, $\exists Z, (X, Y, Z)$ is a SET. (Fun problem: how many distinct SETS are possible in the game SET?)