

1 Quantifiers and Negation

- Negate the following statements. Which is true, the original or the negation? If an existential statement is true, demonstrate it with an example. If a universal statement is false, demonstrate it with a counterexample.
 - $\forall x \in \mathbb{Z}, x > 3$.
 - $\forall x \in \mathbb{Z}, 0x = 0$.
 - $\exists x \in \mathbb{Z}, x$ is prime and negative.
 - $\exists x \in \mathbb{Z}, x = -x$.

2 Sets and Quantifiers

In this section, consider the set $S = \{1, 2, \{2\}, 3, \{1, 2\}, \{4\}\}$.

- For each statement, agree on what it means, and determine if it is true or false. If it is a false universal statement, give a counterexample. If it is a true existential statement, give the example that makes it true. To make this more fun, circling the letter under the correct column for each statement will spell something.

	TRUE	FALSE
$\exists x \in S, x = 2$	S	G
$\exists x \in S, x = 4$.	A	E
$\forall x \in S, x = 2$.	R	T
$\forall x \in S, x \in \mathbb{Z}$.	T	S
$\forall x \in S, x \subseteq \mathbb{Z}$.	C	A
$\exists x \in \mathbb{Z}, x \in S$.	R	L
$\exists x \in S, x \in \mathbb{Z}$.	E	A
$\exists x \in S, \{x\} \in S$.	T	S
$\forall x \in S, (x \in \mathbb{Z}) \vee (x \subseteq \mathbb{Z})$.	H	O
$\forall x \in \mathbb{Z}, x \notin S$.	O	E
$\forall x \in \mathbb{Z}, \{x\} \notin S$.	D	C
$\forall x \in \mathbb{Z}, \{\{x\}\} \notin S$.	A	O
$\forall x \in S, \{x\} \subseteq S$.	T	G
$\forall x \in \mathbb{Z}, (1 \leq x \leq 4 \implies x \in S)$.	H	S
$\forall x \in \mathbb{Z}, (1 \leq x \leq 3 \implies x \in S)$.	P	D
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \in S)$.	A	J
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \subseteq S)$.	S	W

- Now negate each statement in the list. Double-check that its truth value changes (if it was false, it becomes true, and vice versa).