Optional Proof Revision Opportunity

Math 2001, Spring 2023. Katherine E. Stange.

Doing this assignment can only improve, not lower, your score.

This is an opportunity to improve your proof grade in the course. You may choose one or two proof quizzes that you did in the class, and produce a document explaining your errors, and how to fix them. These can be any proofs upon which you had feedback (whether or not they counted toward your final grade).

Due: At the final exam.

For each revision you will do the following:

1. Include the original quiz including feedback (my comments on your original test), as a scan.
2. Give a number or symbol to each of the distinct comments I gave to you as feedback. (A distinct comment may be several sentences long, or it may be a word or two. Break them up according to **content**: one comment means one concept/idea/error.)
3. For **each of the comments**:
   (a) Make sure you understand the issue I'm commenting on. I'm available for consultation and will happily explain in further detail.
   (b) Write a paragraph in your own words explaining what the error or confusion was and why it was problematic in terms of logic or communication. Try to write as clearly as possible, as if writing a textbook for your peers.
   (c) Demonstrate how to improve this one aspect of the proof. **Do not rewrite the entire proof from scratch.** Instead, focus on this particular error, and how to improve this particular problem, **in the context in which it is found**. For example, show how you can replace the sentence you have a with a better sentence, in the context of your own proof.
4. If you see further improvements you can make, based on your own understanding (even if I didn't comment on them), please include these as if they were my own comments (same process as above).
5. Make sure the final proof (as improved above) is correct.
6. Feel free throughout this process to consult with me and ask if corrections are good, etc. I'm more than happy to talk things over with you.
7. Give the final proof, as improved by each of the comments. **Do not rewrite the entire proof.** The purpose is to demonstrate the contrast/improvement between past and present. The focus is on **process**.

Grading is based on:

1. How carefully you follow these instructions.
2. How convincingly you **demonstrate your understanding of your errors**
3. How clearly you explain the meaning of my comments (as if for a textbook for your peers).
4. The effectiveness of your **process by which you improve your own proof**.
5. The effectiveness/correctness of your corrections.

Choose your proofs to improve based on the grading above. In particular,

1. Choose proofs with lots of specific comments to improve. (Quantity does matter, as you can demonstrate yourself more effectively.)
2. Do not choose proofs that require complete rewriting to be correct, as this doesn't demonstrate *improvement*, it just demonstrates *replacement*.

I will give you a final grade on each proof revision that you do (you may do one or two revisions), and these grades will replace your lowest proof grades currently counting into your class grade, if this is an improvement.

Doing this assignment can only improve, not lower, your score.

Example

Theorem 1. *Let f_n be the n-th Fibonacci number. That is, $f_1 = f_2 = 1$ and $f_{n+2} = f_{n-1} + f_n$ for $n \geq 1$. For all $n \geq 2$, we have $f_n < 2^n$.*

0.1 Original proof

Proof. We will prove this by induction on n.

Base case: Let $n = 2$. Then $f_2 = 1 < 2^2 = 4$.

Inductive step: Suppose the theorem holds for $2 \leq n \leq k$, were $k \geq 3$. We will prove that it holds for $n = k+1$. Using the inductive hypothesis for $n = k$ and $n = k - 1$, we have

$$f_{k+1} = f_k + f_{k-1} < 2^k + 2^{k-1} < 2^k + 2^k = 2^{k+1}.$$

□

0.2 Teacher's comment:

You need more base cases.

0.3 Explanation of comment:

In an inductive proof, all subsequent cases must have a path, by using the inductive step repeatedly, to a base case. In this proof as currently written, our base case is $n = 2$. Our inductive step proves case n in terms of $n - 1$ and $n - 2$. In particular, to prove the theorem for $n = 3$ ($k = 2$ in the language of the proof), the inductive step relies on previous cases $n = 1$ and $n = 2$. However, the theorem is not stated for $n = 1$. Therefore the current structure does not prove the theorem for $n = 3$. Instead, we can include $n = 3$ as a base case. (Alternatively, we could include $n = 1$, as the theorem holds for that also, even though it is not stated there.) Then the case $n = 4$ depends on $n = 3$ and $n = 2$ (which are both base cases); the case $n = 5$ depends on $n = 4$ and $n = 3$ (one shown previously, one a base case); etc.

0.4 Correction:

We will replace the previous base case:

Base case: Let $n = 2$. Then $f_2 = 1 < 2^2 = 4$.

with a more extensive base case:

Base cases: Let $n = 2$. Then $f_2 = 1 < 2^2 = 4$. Let $n = 3$. Then $f_3 = f_2 + f_1 = 1 + 1 = 2 < 2^3 = 8$.

0.5 Complete final proof.

Proof. We will prove this by induction on n.

Base cases: Let $n = 2$. Then $f_2 = 1 < 2^2 = 4$. Let $n = 3$. Then $f_3 = f_2 + f_1 = 1 + 1 = 2 < 2^3 = 8$.

Inductive step: Suppose the theorem holds for $2 \leq n \leq k$, were $k \geq 3$. We will prove that it holds for $n = k+1$. Using the inductive hypothesis for $n = k$ and $n = k - 1$, we have

$$f_{k+1} = f_k + f_{k-1} < 2^k + 2^{k-1} < 2^k + 2^k = 2^{k+1}.$$

□