

Proof Quiz #7 Solution

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Honor Code Rules

Proof Quizzes are open book, but are to be completed on your own without collaboration. To be specific, you may use your course notes, textbook, course website resources, course videos. You may not use the internet beyond the course websites. You may not ask anyone else for help (except your professor), including other humans, or posting/entering your question into the internet. You may not share the questions or answers with anyone else.

Have you read, understood, and followed the honor code rules above?

YES / NO

Please write your best written proof of the following theorem. You will be graded on logic as well as writing.

Theorem 1. *Prove that the sequence $1 + \frac{1}{n^2}$ converges to 1.*

Hints: This is similar to what we did in the first half of class Friday October 23rd.

Proof. Let $\epsilon > 0$. Let N be an integer greater than $1/\sqrt{\epsilon}$. In particular, $N^2 > 1/\epsilon$ and so $1/N^2 < \epsilon$.

Now suppose $n > N$. Then

$$\left| 1 + \frac{1}{n^2} - 1 \right| = \left| \frac{1}{n^2} \right| = \frac{1}{n^2} < \frac{1}{N^2} < \epsilon.$$

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