Math 2001, Spring 2023. Katherine E. Stange.

1 Assignment

Prove the following theorem.

Theorem 1. *If n is a natural number, then*

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

Proof. We will prove this by induction.

Base Case:

Let $n = 1$. Then the left side is $1 \cdot 2 = 2$ and the right side is $\frac{1 \cdot 2 \cdot 3}{3} = 2$.

Inductive Step:

Let $N > 1$. Assume that the theorem holds for $n < N$. In particular, using $n = N - 1$,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + (N-1)N = \frac{(N-1)N(N+1)}{3}$$

Then using the above equation, we compute

$$\begin{aligned}
&1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + N(N+1) \\
&1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + (N-1)N + N(N+1) \\
&= \frac{(N-1)N(N+1)}{3} + N(N+1) \\
&= \frac{(N-1)N(N+1) + 3N(N+1)}{3} \\
&= \frac{N(N+1)(N-1+3)}{3} \\
&= \frac{N(N+1)(N+2)}{3}.
\end{aligned}$$

□