Math 2001: Negation (Katherine Stange, Spring 2023)

There are a collection of people in the room.

Name	Hat colour	Shoe size
Alexa	red	7
Josh	green	5
Amieh	red	1
David	blue	3
Hannah	red	10
Yiting	yellow	17
Connor	blue	7

Task 1: Mark if each statement is true or false.
Task 2: Negate each statement.
Task 3: Check if the negation is true or false.

1. There exists someone with a purple hat in the room.

   No one in the room has a purple hat.

2. There exists a negative integer which is a perfect square.

   No negative integer is a perfect square.

   Negative integers cannot be perfect squares.

   Every negative integer fails to be a perfect square.

3. There exists someone with a blue hat in the room.

   No one in the room has a blue hat.

4. There exists a real number x such that $x = -x$.

   There is no real number x such that $x = -x$.

5. Every shoe size in the room is greater than 0.

   There is some shoe size in the room that is less than or equal to 0.

6. Every integer is a rational number.

   There is some integer which is not rational.

7. All hats in the room are either red or green.

   There is some hat in the room which neither red nor green.

8. All real numbers are either positive or negative.

   There's some real number which is neither positive nor negative.

9. If someone is wearing a red hat, then they have size 7 shoes.

   There is someone wearing a red hat but who does not have size 7 shoes.

   Comment: The if-then statement is a rule. To state it is to claim the rule is being followed. The negation is that the rule is not being followed. So someone is breaking it.

10. If $x \in \mathbb{Z}$ is even, then $x > 3$.

    There is an integer $x \in \mathbb{Z}$ which is even but ≤ 3.

    There is an integer $x \in \mathbb{Z}$ which is even and ≤ 3.

11. If someone is wearing a green hat, then they have size 5 shoes.

    There's someone wearing a green hat who does not have size 5 shoes.

12. If $x \in \mathbb{Z}$ is even, then x^2 is even.

    There an integer x which is even but for which x^2 is odd.

13. If someone is wearing a purple hat, then they have size 7 shoes.

    There's someone wearing a purple hat who does not have size 7 shoes.

14. If $x \in \mathbb{Z}$ is a negative perfect square, then x is prime.

    There exists $x \in \mathbb{Z}$ which is a negative perfect square and not prime.

15. Everyone has a positive shoe size and a red hat.

    Someone either has a non-positive shoe size or does not have a red hat or both.

    Someone either has a non-positive shoe size or does not have a red hat.

    Comment: the 'or both' is understood in mathematics when the word 'or' is used.

16. All integers are rational and prime.

    There exists an integer which is not rational or not prime.

17. Everyone has a positive shoe size and a coloured hat.

    Someone has a non-positive shoe size or does not have a coloured hat.

18. All integers are rational and real.

    At least one integer is not rational or not real.

    There exists an integer which is not rational or not real.

19. Everyone has either a two-digit shoe size or a red hat.

    Someone fails to have a two-digit shoe size and fails to have a red hat.

    There is someone without a two-digit shoe size and without a red hat.

20. All integers are either odd or prime.

    There exists an integer which is both even and not prime.

21. Everyone has either a name beginning with a consonant or an odd shoe size.

    Someone has a name not beginning with a consonant and does not have an odd shoe size.

    There is someone whose name begins with a vowel and has an even shoe size.

22. Every integer greater than 5 is either odd or composite.

    There is some integer greater than 5 which is even and not composite.