Logic Quiz (Katherine E. Stange, Math 2001, Spring 2023, CU Boulder)

Name:

Correct answers without justification will receive full credit (unless justification is required by the question). Incorrect answers with explanation can receive partial credit. If the questions are unclear, please ask during the test and I will clarify.

1. Give the truth table for $P \Rightarrow Q$. This has two variables, so there should be four rows.

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

2. Draw the truth table for $((\sim P) \vee Q) \Rightarrow R$. There are three variables, so this should have 8 rows. In case of partial credit, please include the intermediate columns.

P	Q	R	~P	(~P)∨Q	((~P)∨Q)⇒R
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

3. Simplify the expression until it has *three letters at most* (currently it has four):

$$(P \wedge Q) \vee (P \wedge R)$$

$= P \wedge (Q \vee R)$

(applying a distributivity law)

4. Simplify the expression until it has *no negation symbol*:

$$\sim (P \Rightarrow (\sim Q))$$

$= P \wedge (\sim(\sim Q))$

$= P \wedge Q$

negation of $\Rightarrow$

$\sim(\sim Q) = Q$

5. For each symbolic statement, determine if it is true or false.

   (a) $\forall (x, y) \in \mathbb{R} \times \mathbb{R}, x < y$.

   "All pairs of real x & y satisfy $x<y$" TRUE, FALSE

   the pair $(3,1)$ fails this rule

   (b) $\forall x_1, x_2 \in \mathbb{R}, \exists y \in \mathbb{R}, x_1 = y + x_2$.

   TRUE, FALSE

   for any x_1, x_2, you can solve for y

   (c) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x^2 \neq y$.

   "There's a real x that's not a square." TRUE, FALSE

   -1 is not a square (of a real)

   (d) $\exists X \subseteq \mathbb{Z}, 12 \in X$.

   TRUE, FALSE

   $\{12, 13\}$ is such a set, for example.

6. For each english statement, give the same statement in symbolic notation (**no english words**), including such things as quantifiers ($\exists$, $\forall$), boolean operators ($\sim$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$), etc. Any particular one may or may not need quantifiers.

   (a) There is a negative integer.

   $\exists x \in \mathbb{Z}, x < 0.$

   (b) If $x \in \mathbb{Z}$ is negative, then x is non-zero.

   $\forall x \in \mathbb{Z}, x < 0 \Rightarrow x \neq 0.$

   or: $(x \in \mathbb{Z} \wedge x < 0) \Rightarrow x \neq 0.$

   (c) The empty set is a subset of every subset of $\mathbb{Z}$.

   $\forall X \subseteq \mathbb{Z}, \phi \subseteq X.$

7. Statement: If f is differentiable, then f is continuous.

   - Give the converse: If f is continuous, then f is differentiable.

   - Give the contrapositive:

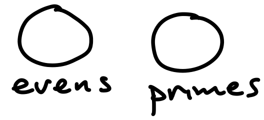
     If f is not continuous, then f is not differentiable.

8. Choose the correct negation for each statement.

   (a) There is an even prime.

      i. There is an odd composite.
      ii. Every prime is even.
      iii. There is an odd prime.
      iv. All odd numbers are prime.
      v. Every prime is odd.
      vi. I have a hamster.

   Another correct answer:
   "Every even # is not prime."
   These are 2 ways to say that <u>evens</u> & <u>primes</u> don't intersect.



   (b) The integer 7 is odd and prime.

      i. The integer 7 is not odd or not prime.
      ii. The integer 7 is not odd and prime.
      iii. The integer 7 is odd or prime.
      iv. The integer 7 is not odd or prime.
      v. The integer 7 is not odd and not prime.
      vi. The integer 7 is odd and not prime.

   (c) If an integer is odd, then it is positive.

      i. If an integer is not odd, then it is not positive.
      ii. There is an odd integer that is not positive.
      iii. If an integer is not positive, then it is not odd.
      iv. All odd integers are non-positive.
      v. All integers are odd and positive.
      vi. If an integer is odd, then it is not positive.

   (d) All functions f such that $f(0) = 0$ have a derivative.

      i. Some functions f such that $f(0) = 0$ do not have a derivative.
      ii. All functions f such that $f(0) = 0$ do not have a derivative.
      iii. All functions f such that $f(0) \neq 0$ have a derivative.
      iv. Some functions f such that $f(0) \neq 0$ have a derivative.
      v. If f is a function such that $f(0) = 0$, then f has a derivative.

9. True or False

   (a) $Q = Q \vee Q$ True
   (b) $\sim (P \Rightarrow Q) = P \vee (\sim Q)$ False $\sim(P \Rightarrow Q) = P \wedge (\sim Q)$
   (c) $P \Leftrightarrow (\sim Q) = (\sim P) \Leftrightarrow Q$ True "P and ~Q agree" = "~P and Q agree"
   (d) $P \Rightarrow Q = (\sim Q) \Rightarrow P$ False $P \Rightarrow Q = (\sim Q) \Rightarrow (\sim P)$
   (e) $\sim (P \vee Q) = (\sim P) \vee (\sim Q)$ False $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$ De Morgan's

10. Consider all possible boolean expressions in two variables ($P \Rightarrow Q$, $P \vee P \vee (\sim P) \vee Q$ etc.). Consider two of them the *same* if they are logically equivalent. How many *different* (logically inequivalent) such expressions are there? Why?

    "logical equivalence" means truth tables agree.
    For 2 variables, truth tables have 4 rows.
    Each row has two options (T/F).
    So there are $2^4 = 16$ possible truth tables.
    So 16 <u>different</u> expressions.