

Induction

April 9, 2020

1 The induction format

Induction breaks one proof into two smaller proofs. For each theorem, write the **statement of** the base case and the inductive step (weak and strong versions). Don't do the proofs, just tell me what you need to prove for each piece. In fact, some of the statements are wrong!

1. Every positive integer is even or odd.
 - (a) **Base Case:** The number 1 is even or odd.
 - (b) **Inductive Step (weak):** For $n > 1$, if $n - 1$ is even or odd, then n is even or odd.
 - (c) **Inductive Step (strong):** If all integers $1 \leq k < n$ are even or odd, then n is even or odd.
2. Any positive number of hamsters is tasty as a treat.
 - (a) **Base Case:**
 - (b) **Inductive Step (weak):**
 - (c) **Inductive Step (strong):**
3. Any non-negative integer can be written as a sum of four squares.
 - (a) **Base Case:**
 - (b) **Inductive Step (weak):**
 - (c) **Inductive Step (strong):**
4. $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ for any positive integer n .
 - (a) **Base Case:**
 - (b) **Inductive Step (weak):**
 - (c) **Inductive Step (strong):**
5. Any non-empty finite set of hamsters is enough hamsters to warm your heart.
 - (a) **Base Case:**
 - (b) **Inductive Step (weak):**
 - (c) **Inductive Step (strong):**
6. Any tree on $n \geq 2$ vertices has at least 2 leaves.
 - (a) **Base Case:**
 - (b) **Inductive Step (weak):**
 - (c) **Inductive Step (strong):**