1 The induction format ("strong")

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For each theorem, envision it as falling into cases that are parametrized by some integer. Imagine how larger cases can be proven from knowledge of smaller cases. Induction breaks one proof into two smaller proofs: the Base Case and the Inductive Step. This exercise asks you to state what is needed to prove for each part.

Don't do the proofs, just tell me what you need to prove for each piece. In fact, some of the statements are wrong!

The first few are done for you. See if you can understand the pattern and fill in the missing ones.

1. If T is a tree, then it has one fewer edge than it has vertices.
   (a) **What are we inducting on?** The number of vertices.
   (b) **Base Case:** Prove that the tree on 1 vertex has 0 edges.
   (c) **Inductive Step (strong):** Let $n > 1$. Suppose any tree with $1 \leq k < n$ vertices has $k - 1$ edges. Prove that any tree with n vertices has $n - 1$ edges.

2. Every positive integer is even or odd.
   (a) **What are we inducting on?** The integer.
   (b) **Base Case:** Prove that the number 1 is even or odd.
   (c) **Inductive Step (strong):** Let $n > 1$. Suppose all integers $1 \leq k < n$ are even or odd. Prove n is even or odd.

3. Any positive number of hamsters is tasty as a treat.
   (a) **What are we inducting on?** The number of hamsters.
   (b) **Base Case:** Prove that one hamsters is tasty as a treat.
   (c) **Inductive Step (strong):** Let $n > 1$. Suppose any positive number of hamsters fewer than n is tasty as a treat. Prove that n hamsters are tasty as a treat.

4. Any non-negative integer can be written as a sum of four squares.
   (a) **What are we inducting on?**
   (b) **Base Case:**
   (c) **Inductive Step (strong):**

5. Any non-empty finite set of hamsters is enough hamsters to warm your heart.
   (a) **What are we inducting on?**
   (b) **Base Case:**
   (c) **Inductive Step (strong):**
6. Any tree on $n \geq 2$ vertices has at least 2 leaves.
   (a) **What are we inducting on?**
   (b) **Base Case:**
   (c) **Inductive Step (strong):**
7. $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for any positive integer n.
   (a) **What are we inducting on?**
   (b) **Base Case:**
   (c) **Inductive Step (strong):**