Example sets problems (Katherine E. Stange, Math 2001, CU Boulder)

1. Let $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3, 4\}$ and $C = \{2, 3\}$.
   (a) Compute $A \cup B$.
   (b) Compute $A \cap B$.
   (c) Compute $A - B$.
   (d) Compute $A - C$.
   (e) Compute the complement of A in the universe $\{1, 2, 5, 6, 7\}$.
2. Let A, B and C be arbitrary sets. Draw a Venn diagram illustrating $(A \cap B) \cup C$.
3. If the complement of $\{a, b\}$ in the universe S is $\{e, f\}$, then what is S?
4. Let $A = \{a, b, c, d, e, f\}$ and $B = \{d, e, f, g, h, i\}$ and $C = \{a, b, c, d, e, f, g, h, i, j, k, l\}$.
   (a) Compute $A \cup B$.
   (b) Compute $A \cap B$.
   (c) Compute $A - B$.

(d) Compute $A - C$.

(e) Compute the complement of A in the universe C.

5. Draw a Venn diagram illustrating $(A \cup B) \cap C$.

6. What is the complement of the set of even integers with respect to the set of all integers?

7. Compute the power set $\mathscr{P}(\{b, \{1, 2\}\})$.

8. Compute the Cartesian product $A \times A \times A$ where $A = \{1, \emptyset\}$.

9. Draw a picture (on the real plane) of the Cartesian product $[1, 2) \times [0, 2]$.

10. Is the following region of the plane a Cartesian product? If yes, gives sets A and B so that it is $A \times B$.

$$YES, \quad NO$$

$$\text{If yes, } A = $$
$$B = $$

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$$YES, \quad NO$$

$$\text{If yes, } A = $$
$$B = $$

12. Suppose that A and B are sets satisfying $|\mathscr{P}(A) \times B| = 12$, and B has fewer than 4 elements. Then what are the cardinalities of A and B?

$$|A| =$$
$$|B| =$$

13. Mark each of the following as TRUE, FALSE or NOT WELL-DEFINED (for example, if the set builder notation isn't valid – watch out for these!):

    (a) $1 \in \{x \in \mathbb{R} : x^2 + 1\}$

    TRUE, FALSE, NOT WELL-DEFINED

    (b) $1 \in \{x^2 + 1 : x \in \mathbb{R}\}$

    TRUE, FALSE, NOT WELL-DEFINED

    (c) $(-\infty, 4] \subseteq (1, 8)$

    TRUE, FALSE, NOT WELL-DEFINED

    (d) $\{(x, x) : x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$

    TRUE, FALSE, NOT WELL-DEFINED

    (e) $\{\sin(x) : x \in \mathbb{R}\} = [-1, 1]$

    TRUE, FALSE, NOT WELL-DEFINED

    (f) $\{1, 2, 3\} \in \{A \subseteq \mathbb{Z} : \{1, 2\} \subseteq A\}$

    TRUE, FALSE, NOT WELL-DEFINED

    (g) $|\{x \in \mathbb{R} : x^2 = 3\}| = 2$

    TRUE, FALSE, NOT WELL-DEFINED

14. Give set builder notation for the set of real numbers whose squares are larger than 7. Use symbols exclusively (not english words).

15. Give set builder notation for the upper half plane, i.e. the part of the Cartesian plane lying above the x-axis (not including the x-axis). Use symbols exclusively (not english words).

16. Give set builder notation for set of subsets of the integers that include 0 (for example, $\{0, 1, 2\}$ is in this set, but $\{1, 2\}$ is not). Use symbols exclusively (not english words).

17. Let $A = \{1, \emptyset, \{3\}\}$ and $B = \{1, 2\}$.

    (a) Compute the power set of A.

(b) Compute the Cartesian product $A \times B$.

(c) Compute the Cartesian product $B \times \emptyset$.

(d) Compute the following cardinality (not the set itself):

$$|\mathscr{P}(A \times B)| =$$

18. Let $|A| = n$ and $|B| = m$. What is the cardinality of $\mathscr{P}(A^k) \times B$?

19. For each set, determine if it is a Cartesian product. If it is, then find the two sets A and B so that it is $A \times B$. If it is not, state clearly that it is not.

(a) $\{(1,a),(1,b),(2,a),(2,b),(2,c)\}$

(b) In $\mathbb{R}^2$, the square with vertices $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$.

(c) In $\mathbb{R}^2$, the triangle with vertices $(0,0)$, $(0,1)$ and $(1,0)$.

20. Mark each of the following as TRUE, FALSE or NOT WELL-DEFINED (for example, if the set builder notation isn't valid – watch out for these!):

(a) $2 \in \{x \in \mathbb{Z} : 2x\}$

TRUE, FALSE, NOT WELL-DEFINED

(b) $2 \in \{2x : x \in \mathbb{Z}\}$

TRUE, FALSE, NOT WELL-DEFINED

(c) $1/2 \in \{x^2 : x \in \mathbb{R}\}$

TRUE, FALSE, NOT WELL-DEFINED

(d) $\{(x,x^2) : x \in \mathbb{R}\} \subseteq \{(x,y) : x \in \mathbb{R}, y \in \mathbb{Z}\}$

TRUE, FALSE, NOT WELL-DEFINED

(e) $\{\cos(x) : x \in \mathbb{R}\} = \{\sin(x) : x \in \mathbb{R}\}$

TRUE, FALSE, NOT WELL-DEFINED

(f) $(3,4] \subseteq (3,4)$

TRUE, FALSE, NOT WELL-DEFINED

(g) $|\{x \in \mathbb{R} : x^2\}| = 2$

TRUE, FALSE, NOT WELL-DEFINED

21. Give set builder notation for the set of positive rational numbers. Use symbols exclusively (not english words).

22. Give set builder notation for the set of subsets $\mathbb{Z}$ which contain the element 0. Use symbols exclusively (not english words).

23. Give set builder notation for your favourite parabola, living in the Cartesian plane. Use symbols exclusively (not english words).

24. Give an example of set builder notation that represents the set $\{1\}$. Use symbols exclusively (not english words). Note: $\{1\}$ is set notation, but is not set builder notation. Set builder means you have a colon in there somewhere, among other things.

25. Give an example of a set A which *simultaneously* satisfies all the following things: (a) $|A| = 3$; (b) $2 \in A$; and (c) $\{1\} \subseteq A$.

26. Let $A = 1$ and $B = \{1\}$. Circle the true fact(s):

$$A = B, \quad A \subseteq B, \quad A \in B, \quad B \subseteq A, \quad B \in A, \quad |A| = 1, \quad |B| = 1$$

27. Let $A = \{\{1\}\}$ and $B = \{\{1\}, 1\}$. Circle the true fact(s):

$$A = B, \quad A \subseteq B, \quad A \in B, \quad B \subseteq A, \quad B \in A, \quad |A| = 1, \quad |B| = 1$$

28. Is the empty set a subset of every set? Explain why or why not.

29. Is the empty set an element of every set? Explain why or why not.

30. What is the cardinality of $\{\mathbb{R}\}$?

31. Give an example of a subset of $\{1, 3, 5, 7, 9\}$.

32. Give an example of an element of $\{\mathbb{Z}, \mathbb{N}, \mathbb{R}, \mathbb{Q}\}$.

33. What is the cardinality of $\emptyset$?

34. What is the cardinality of $\{\emptyset\}$?

35. Circle those of the following which are TRUE:

    (a) $\emptyset \subseteq \{3, 4\}$

    (b) $\emptyset \in \{3, 4\}$

    (c) $1 \subseteq \{1\}$

    (d) $1 \in \{1\}$

    (e) $|\{1, 3, 5\}| = 3$